

Reliability for $U[0,10]$

$$f(x) = .1 \text{ in } 0..10$$

$$F(x) = x/10 \text{ for } 0 \leq x \leq 10$$

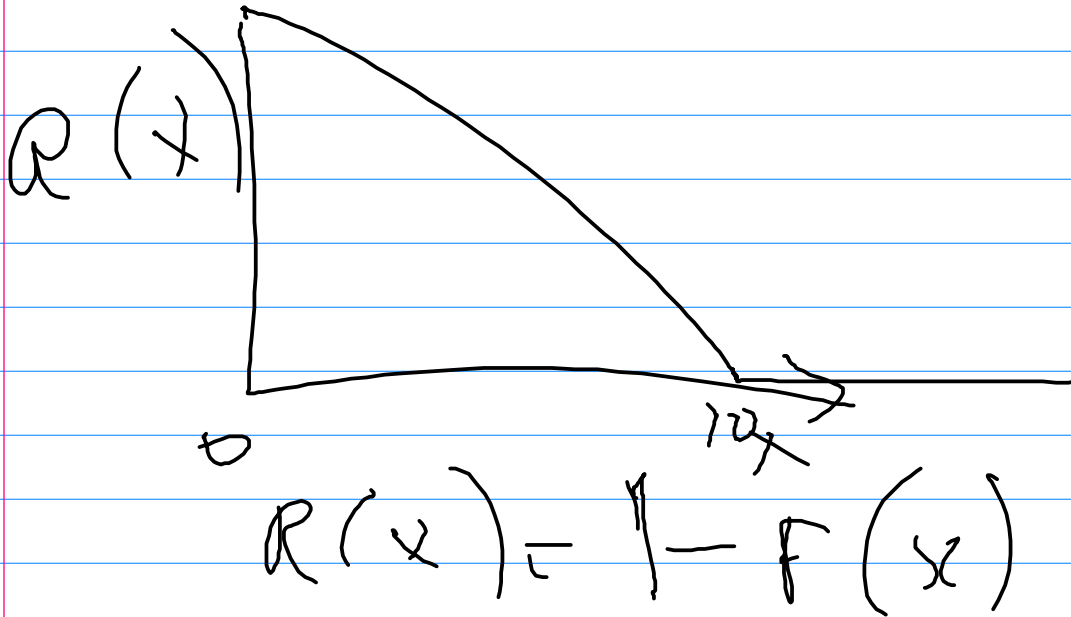
$$0 \text{ for } x < 0$$

$$1 \text{ for } x > 10$$

$$R(x) = 1 - x/10$$

$$R(5) = .5$$

$$R(9) = .1$$



$f(x) dx = \text{prob dies between time } x \text{ and time } x+dx \text{ for small } dx$

event: object dies

$$P[\text{dies between 1 and 2}] = f(x) dx = .1 * 1 = .1$$

$$P[\text{dies between times } a \text{ and } b] =$$

$$\int_a^b f(x) dx$$

Max of 2 r.v.

Your car has 2 headlights. You can drive at night so long as at least one works. Analyze this. What's the prob dist of the max life?

r.v. of 1st headlight dies is $U[0,100]$. Call that r.v. X .

Y is r.v. for 2nd.

define $Z = \max(X, Y)$.

def of $F_Z(z) = P[Z < z] = P[\max(X, Y) < z] = P[X < z \text{ \& } Y < z]$

assuming two bulbs are independent

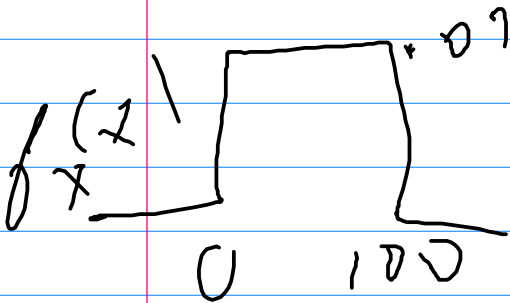
$$P[X < z \text{ \& } Y < z] = P[X < z] P[Y < z] = F_X(z) F_Y(z)$$

$$F_Z(z) = F_X(z) F_Y(z)$$

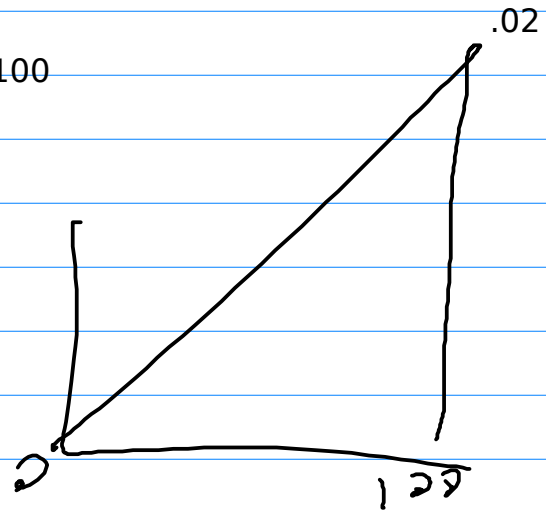
$$f_X(x) = .01 \text{ so } F_X(x) = x/100 \text{ for } 0 < x < 100 \\ = F_Y(x)$$

$$F_Z(z) = z^2 / 10000 \text{ for } 0 < z < 100$$

$$f_Z(z) = z/5000 \text{ for } 0 < z < 100$$



\sqrt{z}

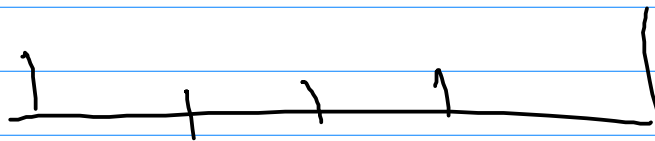


uppercase P means probability. Uppercase E means expected

$$E[Z] =$$

$$\int_0^{100} \frac{z^2}{5000} dz = \left. \frac{z^3}{15000} \right|_0^{100}$$

$\leftarrow 67$



define Z to be max of N uniform r.v. $U[0,1]$

$$F_Z(z) = z^N$$

$$f_Z(z) = N \cdot z^{(N-1)}$$

$$E[Z] = N/(N+1)$$

I have a bulb and a spare. I use the 1st until it fails than swap in the 2nd and use it until it fails. Then start worrying about the grue.

X, Y are r.v. for two bulbs. Define $Z = X+Y$. X, Y defined to be $U[0,1]$.
What is $f_Z(z)$?

Aside: let r.v. be discrete. Each r.v. is # heads for 1 coin toss.
That's Bernoulli.

Z = sum of N coins. That's Binomial. /aside

$$f_Z(z) = \int_0^z f_X(x) f_Y(z-x) dx$$

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$$F_Z(z) = P[X+Y \leq z]$$

suspend this until next class. I'll get a nice motivating example.

failure rate: assume X: $U[0,1]$.