

2/20/2018 p1

iclicker 2: using binomial

$p = \text{prob bad} = .001 = 1000/1000000$

$n = 5$

$5 \ll 1000000$  so ignore that it's selection w/o replacement

$\text{prob}(k \text{ bad of } 5) = (5 \text{ choose } k) p^k q^{(5-k)}$

$\text{prob}(0 \text{ bad}) = 1 \cdot .999^5 = .995$

$\text{prob}(1 \text{ bad}) = (5 \text{ choose } 1) p^1 q^4 = 5 * .001 * .996 = \text{very small}$

$\text{prob}(1 \text{ bad})$  is much less than  $\text{prob}(0 \text{ bad})$  so ignore it.

$\text{prob}(0 \text{ or } 1 \text{ bad}) = .995$

approximation I used:  $(1-e)^n \text{ approx} = 1-ne$  if  $ne \ll 1$

$.999^{10} = .990$  approx

$(1-e)^n = 1 -ne + (n \text{ choose } 2)e^2 - \dots$

Try the same question using Poisson.

The event is the number of bad widgets.

$p = .001$

$\text{Prob}(k \text{ bad}) = p^k e^{-p} / k!$

$\text{Prob}(0 \text{ bad}) = 1 * (1-p) * 1 = .999$

$\text{Prob}(1 \text{ bad}) = .001 * .999 * 1 = .000999 \ll .999$

$p$  here is  $\lambda$  in wikipedia or  $\alpha$  in the text.

Poisson good when there are many possible events, but prob of any one is very small, so expected number is reasonable.

e.g. radioactive decay.

Perhaps there are  $10^{24}$  atoms. Prob of any one decaying is  $10^{-20}$ .

Expected number of decays is 10,000.

book 3.51 a

WANT  $(a+b+c)^n$ 

$$\text{KNOW } (a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$$

DU

$$a+b+c = (a+b) + c$$

$$(a+b+c)^n = ((a+b) + c)^n$$

$$= \sum_{k=0}^n \binom{n}{k} (a+b)^k c^{n-k}$$

$$= \sum_{k=0}^k \binom{k}{u} a^u b^{k-u} c^{n-k}$$

$$= \sum_{k=0}^n \sum_{u=0}^k \binom{n}{k} \binom{k}{u} a^u b^{k-u} c^{n-k}$$

$$\frac{n!}{k!(n-k)!} \cdot \frac{k!}{u!(k-u)!}$$

$$\frac{n!}{(n-k)! \cdot 1! \cdot (k-u)!}$$

$$(a+b)^n = \sum_{k=0}^n \sum_{l=0}^k \frac{n!}{(a-k)! l! (k-l)!} a^{n-k} b^l$$

LET

$$\sum_{k=0}^n \sum_{j=0}^{k-i} \frac{n!}{(n-k)!} a^k b^{k-j}$$

$q = k - j$

q 3.91 on page 139.

Poisson mean  $\lambda$

$$P(k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$k!$

$$P(k|YES) = \frac{\lambda_1^k e^{-\lambda_1}}{k!}$$

$$P(k|NO) = \frac{\lambda_0^k e^{-\lambda_0}}{k!}$$

$$P(YES) = p$$

$$P(k) = P(k|YES) P(YES) +$$

$$P(k|NO) P(NO)$$

$$= \frac{\lambda_1^k e^{-\lambda_1}}{k!} p + \frac{\lambda_0^k e^{-\lambda_0}}{k!} q$$

$$P(YES|k) P(k) = P(YES + k)$$

$$P(Y_{ES} | K) = \frac{\frac{\lambda_1^k}{k!} e^{-\lambda_1} P}{\frac{\lambda_1^k}{k!} e^{-\lambda_1} P + \frac{\lambda_0^k}{k!} e^{-\lambda_0} Q} \quad 5$$

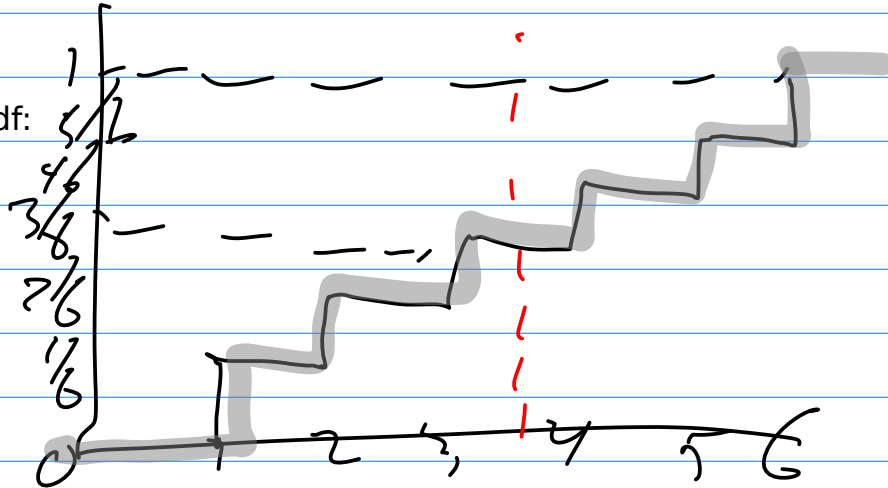
$$= \left( \frac{\lambda_1}{\lambda_0} \right)^k e^{-\lambda_1} P$$

$$\frac{\left( \frac{\lambda_1}{\lambda_0} \right)^k e^{-\lambda_1} P + \left( e^{-\lambda_0} \right) Q}{\left( \frac{\lambda_1}{\lambda_0} \right)^k e^{-\lambda_1} P + \left( e^{-\lambda_0} \right) Q}$$

$$\frac{\left( \frac{\lambda_1}{\lambda_0} \right)^k e^{-\lambda_0 - \lambda_1} P}{\left( \frac{\lambda_1}{\lambda_0} \right)^k e^{-\lambda_0 - \lambda_1} P + Q}$$

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for die cdf:



$$P(\text{DIE AT WAST } \geq \frac{3}{2}) = \frac{1}{2}$$
$$P(\dots, 1) = \frac{1}{6}$$
$$P(\dots, 100) = 1$$

