Conditional probability ctd

Tossing a die
C = we saw an even number {2,4,6}
x= we saw 2
P[x=2] = 1/6
P[C] = 1/2
we want P[2 given C] = 1/6 / (1/2) = 1/3 using 3.26
We need prob we got a 2 and C = 1/6

Motivating ex 3.24: Prob that a light bulb that is already 100 hours old will live another 10 hours.
Human life expectancy: Live expect. given that you've gotten to 20 years.

annotating eqn 3.27 in context of example 3.28

$$P_X(x_k \mid C) = \frac{P_X(x_k)}{P[C]}$$

big X: name of prob distn. uniform [1,L]
little x sub k: particular experiment's outcome
C is the event for a subset of possible outcomes.

In this example, C is the event that the widget is still alive after m units.
xk is outcome that its lifetime is j more, or m+j total.

I want the prob that widget will live another j units if (given) that it's still alive after m.

P[x] = 1/L if 1<=x<=L uniform

PX(xk) = 1/L is 1<=xk <= L

It's X sub k because we're running this experiment k times.
P[C] = (L-m)/L

because its lifetime is m, m+1, m+2, ... L
(perhaps off by 1 error; ignore for now).

p(xk|C) = 1/L / ((L-m)/L) = 1/(L-m)

another example: prob of a random having a baby in lifetime unconditionally and conditioned on being female.
p[have baby] = .4 (numbers made up)
p[female] = .5
p[have baby| female] = .4/.5 = .8
p[baby and female] = .4

2 outcome: baby or no. Bernoulli r.v.

Take all females born in some year, say 1840, look at average year they have first kid. Say 1858.
Take all females having 1st kid in 1858. What's average year they were born? Not 1840. Think.

Variance of r.v.

Bernoulli

outcomes are 1 w.p p, 0 w.p. q=1-p

E[x] = 1 p + 0 q= p
VAR[x] = E[x2]- E[x]2 = 1 p - 0 q - p^2 = p - p^2 = p (1-p) = pq

spread biggest at p=1/2

Binomial r.v. p 115

$$P(X=k) = \binom{n}{k} p^k q^{n-k}$$

$$E(X) = \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k}$$

$$k\binom{n}{k} = \frac{k\, n!}{(n-k)!\, k!\, (k-1)!}$$

$$= n \frac{(n-1)!}{(n-k)!\,(k-1)!}$$

$$= n \binom{n-1}{k-1}$$

$$E(X) = \sum_{k=0}^{n} n \binom{n-1}{k-1} p^k q^{n-k}$$

$$= np \sum_{k=1}^{n} \binom{n-1}{k-1} p^{k-1} q^{n-k}$$

in sum k (n choose k) p^k q^(n-k)

first k is the value of an outcome.
rest is its probability.

also the probs sum to 1

$$\sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} = 1$$

Since expected values sum E[X+Y] = E[X] + E[Y]

and E[one toss] = p
E[n tosses] = p + p + p .... = np for Binomial
expected value for number of heads from n coin tosses.
Coin may be unfair.

Expectations always sum, even for correlated variables.
Variances sum for independent variables.

If variables are dependent... anything can happen.

1st r.v is X. 2nd r.v. is Y=-X.

Z=X+Y. VAR[Z]= 0 because z is always 0.

Other way, let Y=X. Z=X+Y

VAR[Z] = 4 VAR[X].

Poisson: expected number of radioactive decays in
a block of radium in next second.
exp number of hits on web server in next second (if
all potential clients are independent)

e.g. perhaps 10^23 atoms, prob of each one decaying
in next second is 10^(-23).

expected number of decays is 1/second, but it varies.

$$P_k = \frac{\alpha^k}{k!} e^{-\alpha}$$

alpha is the only parameter. Its the mean also variance.

$$\sum_{k=0}^{\infty} P_k = 1$$

$$\sum_{k=0}^{\infty} \frac{\alpha^k}{k!} e^{-\alpha}$$

$$e^{-\alpha} \sum_{k=0}^{\infty} \frac{\alpha^k}{k!}$$

$$= e^{\alpha}$$

DEFINITION

POISSON MEAN

$$E[X] = \sum_k x_k P[X_k]$$

$$= \sum_{k=0}^{\infty} k \frac{\alpha^k}{k!} e^{-\alpha}$$

$$= \alpha \sum_{k=1}^{\infty} \frac{\alpha^{k-1}}{(k-1)!} e^{-\alpha}$$

$$= \alpha e^{-\alpha} \sum_{k=1}^{\infty} \frac{\alpha^{k-1}}{(k-1)!} e^{\alpha}$$

$$= \alpha$$

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$$\sum_{k=0}^{\infty} \frac{\alpha^{k+1}}{(k+1)!}$$

LET $L = k+1$, $k = 0 \Rightarrow L = 1$

$$\sum_{L=1}^{\infty} \frac{\alpha^{L}}{L!} = e^{\alpha} - 1$$

CASE OF $k=1$ CAN BE IGNORED.

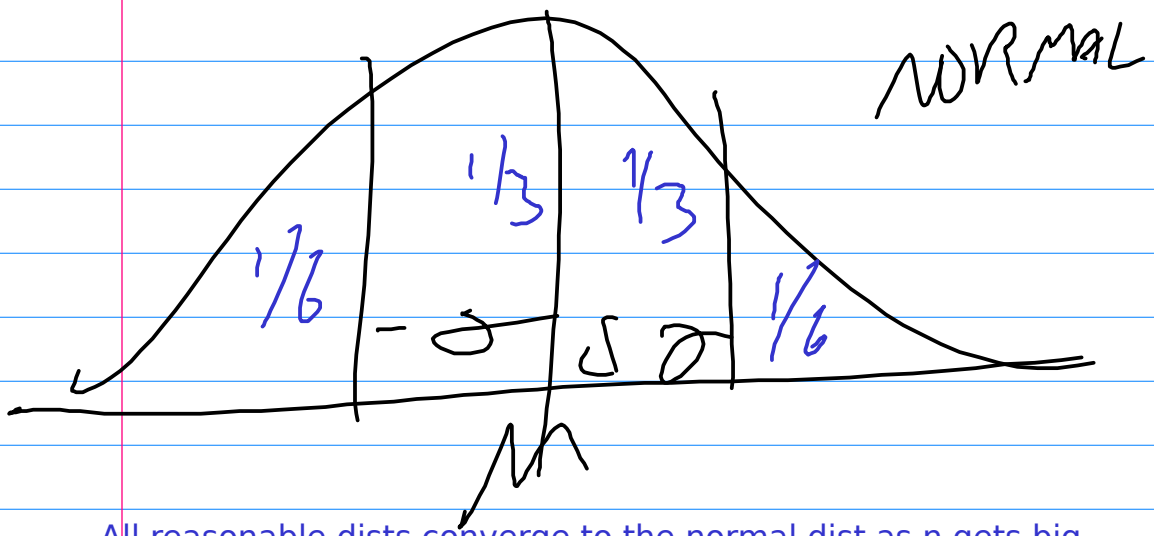
Poisson ctd

E[x] = a, VAR = a STD = sqrt(a)

approx: P[E-std<=x <=E+std] = 2/3

For Poisson, this central interval w.p 2/3 is
[a-sqrt(a), a+sqrt(a)]

As a proportion, it gets smaller as a gets bigger.



All reasonable dists converge to the normal dist as n gets big.
It happens surprisingly quickly.
def "reasonable": dists for which this is true.
dists with finite moments

Binonial dist starts looking like a Poisson for large n
and fixed np =a .
Poission starts looking like a normal dist for large n.
Expected time between consecutive decays is geometric r.v.

Poisson scales:
If the expected number of decays in 1 second is 5,
then expected number in one minute is 5*60=300.