

Numbers are items in my wiki.

1 random experiment: toss coin  
 outcomes: {H,T}  
 assign a number to each outcome.  
 Call that a random variable.  
 H-> 1 T->0

2 outcomes: { TTT, TTH, THT, ...}  
 $X(\text{TTT}) = 0$   $X(\text{TTH}) = 1$  ....  
 from 1st experiment (3 tosses) $x=0$   
 next:  $x=3$   
 next:  $x=1$  ..

3  $Y(\text{HHH}) = 8$   $Y(\text{HTH}) = 2$   $Y(\text{HTT}) = 0$

6 Outcomes have probs.  $p(\text{HTT}) = 1/8$

Random variable has probs.

X: # heads

$$P(X=0) = 1/8$$

$$P(X=1) = 3/8$$

$$P(X=2) = 3/8$$

$$P(X=3) = 1/8$$

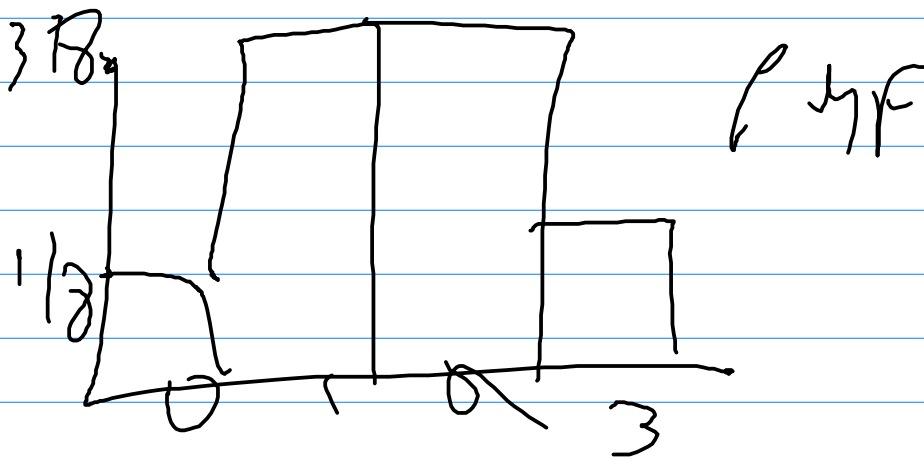
prob mass function (PMF)

Can now do math. E.g. expected value.

$$0 \cdot 1/8 + 1 \cdot 3/8 + 2 \cdot 3/8 + 3 \cdot 1/8 = 1.5$$

10

probs



possible values of random var

12

another uniform random var: toss a 6-sides die

$$p(1) = 1/6 \quad p(7)=0 \quad p(3)=1/6$$

uniform integer from 1 to 6

13

Bernoulli: unfair coin.

2 choices. prob p.

14.1  $p_X(k)$ : probability that the random var  $X$  has value  $k$

In this particular case,  $X$  is a random variable following a geometric distribution with parameter  $p$ .

The random variable  $X$  in 14.1 is unrelated to  $X$  in 12.3 for example.

Geometric dist example. Fair coin  $p=1/2$   
 $q$  defined =  $1-p$

$$p_X(k) = q^{(k-1)} p = 2^{-(k)}$$

$$p_X(1) = 1/2$$

$$p_X(2) = 1/4$$

$$p_X(10) = 1/1024$$

$X$  is the random variable that gives the number of tosses until the 1st head.

another geometric dist example. black pixels on page  
 $Y$  = random var for number of pixels along line until  
 1st black pixel.

App: run-length encoding for fax.

$$p = .01$$

$$Y(k) = q^{(k-1)} p = .99^{(k-1)} * .01$$

$$p_Y(1) = .01$$

$$p_Y(2) = .99 * .01 = .0099$$

In these examples, geom probs decrease towards 0.

15 binomial r.v.

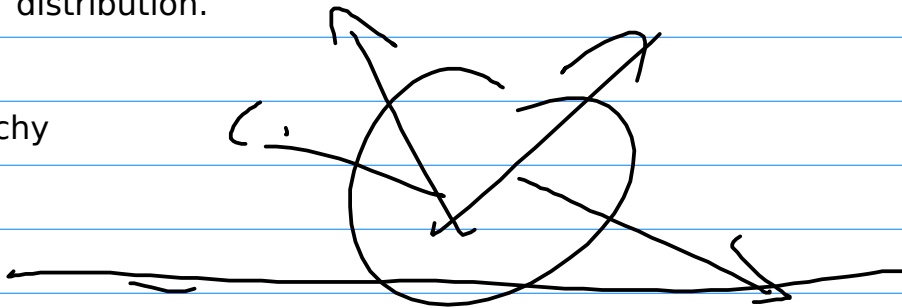
ex. toss coin twice  $n=2$   $p=1/2$

here  $X$  is r.v. of # heads from 2 tosses.

$$p_X(1) = \binom{2}{1} p^1 q^1 = 1/2$$

16 as  $n$  increases graphs becomes similar a 'normal' distribution.

Cauchy



spin a spinner. r.v. is where the projected pointer hits x-axis.

$$p(x) = c / (1+x^2) \quad (\text{I think})$$

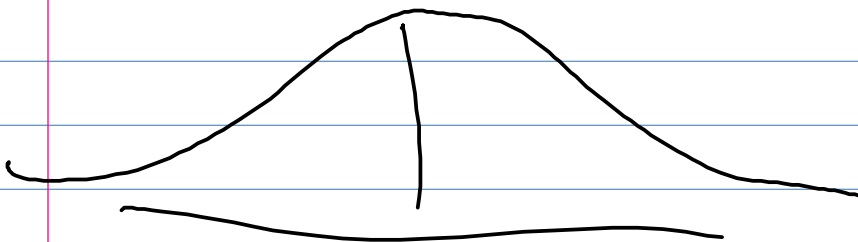
integral is 1. always nonnegative.

however

$$\int \frac{x}{1+x^2} dx$$

diverges.

Cauchy dist does not have a mean or any other moment.



tails don't converge to 0 quickly enough.

Physical meaning of not having a mean. This is getting ahead a little but it's important.

Think about tossing coin  $n$  times and recording number of heads. About  $n/2$  but fluctuates every time you toss  $n$  coins.

Toss coin 10 times, and do this several times.

# heads fluctuates around 5.

Next, toss coin 100 times and repeat.

# heads fluctuates around 50. But, as a fraction of 100, these observations are more tightly clustered around 50.

The larger  $n$  gets, the smaller the percentage fluctuation from the mean is.  
called a law of large numbers.

NOT TRUE for Cauchy.

Perhaps not true for stock market.  
Tails of distribution are too fat.

St Petersburg paradox: bet until you win,  
doubling each time. When you win, that win  
pays all previous losses and still gives a profit.  
What's wrong?

18 Mean of Bernoulli, for  $p=1/2$   
 $p(0) = p(1) = 1/2$  mean =  $1/2*0 + 1/2*1 = 1/2$

Mean of uniform [1.6]:  
 $p(1) = p(2) = 1/6$   
mean =  $1/6*1 + 1/6*2 \dots = 3.5$

19 Mean of binomial for  $n=3$ ,  $p = .1$   
mean =  $(3 \text{ choose } 0) .9^3 * 0$   
 $(3 \text{ choose } 1) .9^2 .1 * 1 +$   
 $(3 \text{ choose } 2) .9 .1^2 * 2$   
 $+ (3 \text{ choose } 3) .1^3 * 3$   
 $= 0 + .243 + .054 + .003 = .3$  (makes sense)

general rule for binomial. mean =  $np$

25 do Mon.