

(6 10/5/17 p1

3D ROTATION

AXIS \hat{a} $|\hat{a}| = 1$

ANGLE Θ

3D POINT $(1, 2, 3) \rightarrow P = 1i + 2j + 3k$

AXIS: X-AXIS $(1, 0, 0)$

$\Theta = 180^\circ$

$$P' = \underbrace{\cos \frac{\Theta}{2}}_0 + \underbrace{\sin \frac{\Theta}{2}}_1 (a_x i + a_y j + a_z k)$$

$\Theta^* = -1$

$$P' = P P P^* = 1 P (-1) = -1 P i'$$

$$P' = -1 (1i + 2j + 3k) i'$$

$$= - (1^2 + 2^2 j + 3^2 k) i'$$

$$= - (-1 + 2k - 3j) i'$$

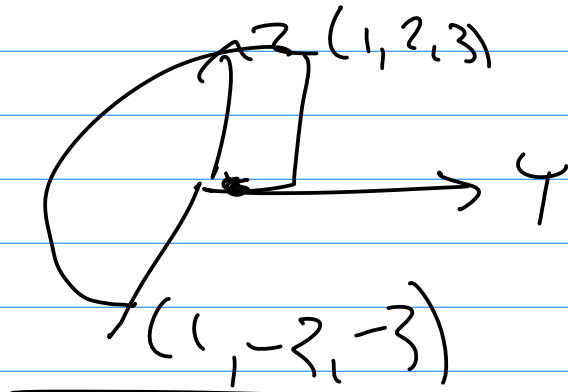
$$= (1 - 2k + 3j) i'$$

$$= (1 - 2k + 3j) i'$$

$$= i - 2j - 3k$$

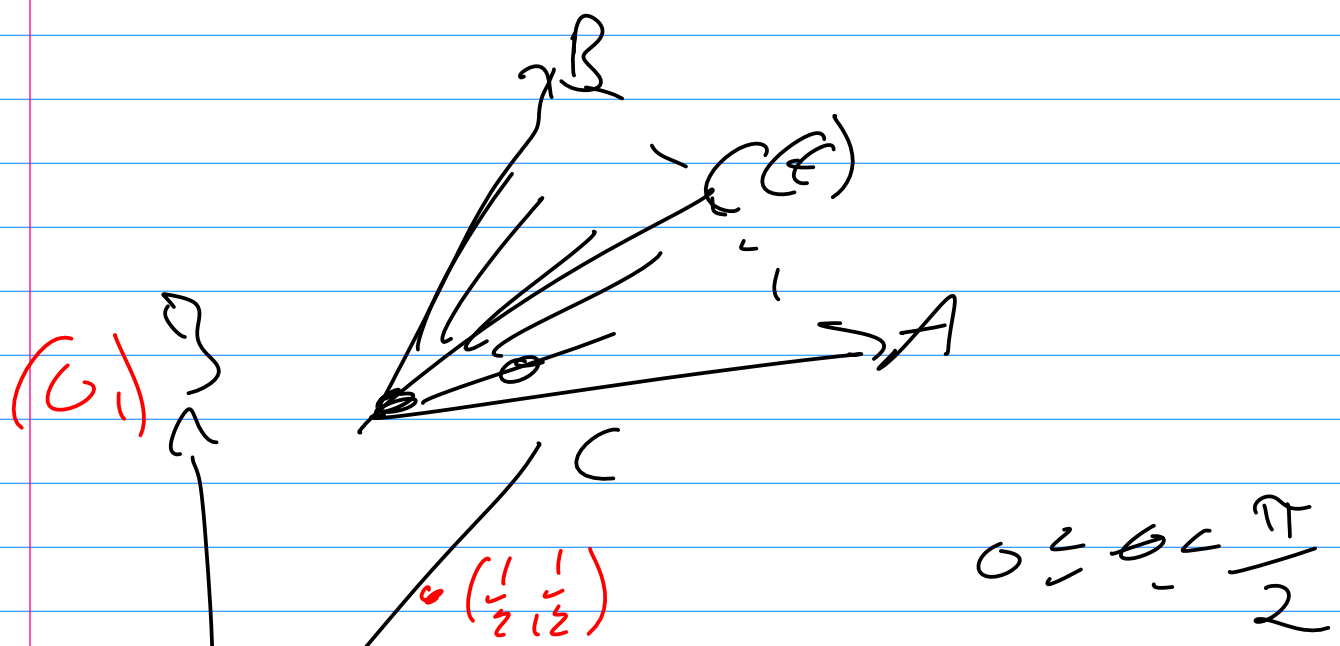
POINT $(1, -2, -3)$

CLAIM ROTATING $(1, 2, 3)$ ²
BY 180° ABOUT Y-AXIS
GIVES $(1, -2, -3)$.



3D INTERPOLATION

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$$C = \frac{(\frac{\pi}{2} - \theta)A + \theta B}{\pi/2}$$

COOL?
NO!
BAD!

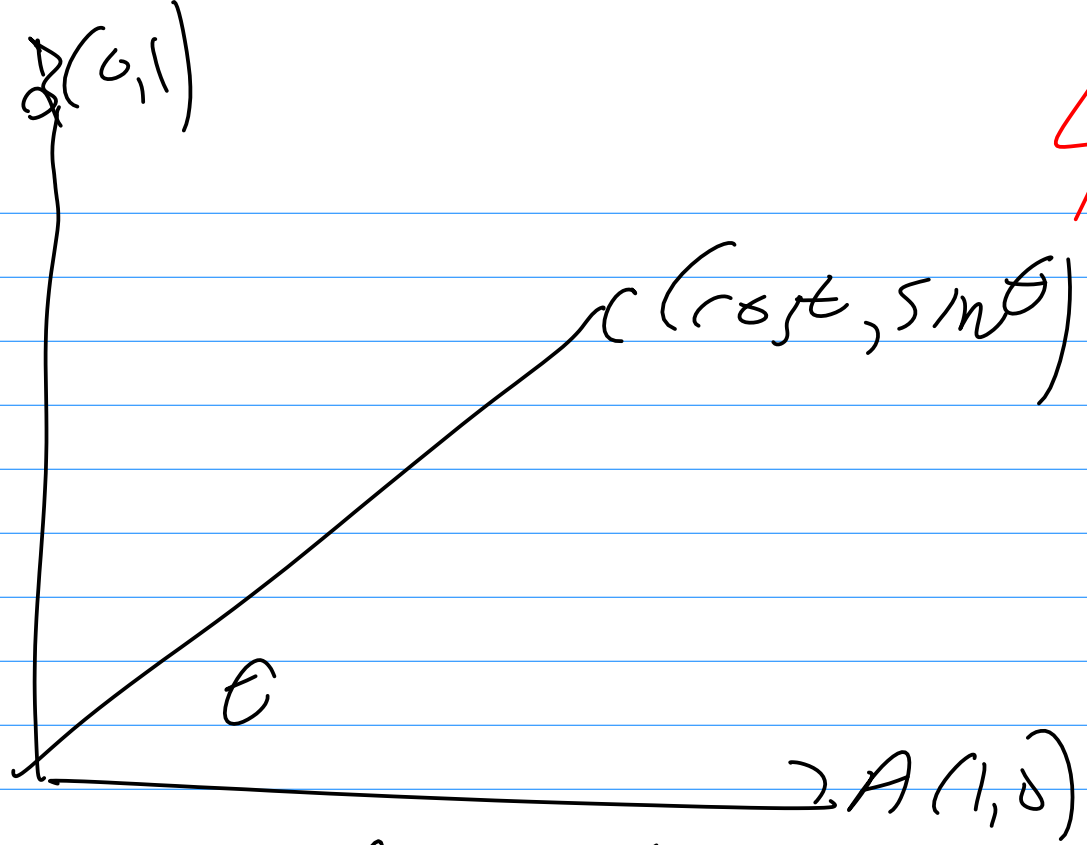
$\theta = 0$ $C = A$

$\theta = \frac{\pi}{2}$ $C = B$

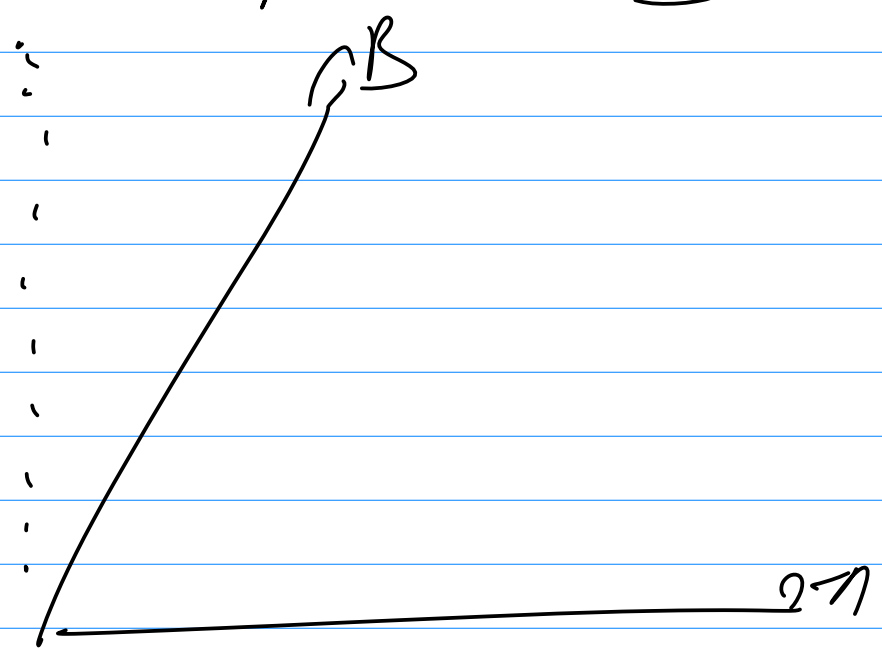
$\theta = \frac{\pi}{4}$ $C = \frac{\frac{\pi}{4}A + \frac{\pi}{4}B}{\pi/2} = \frac{A+B}{2}$ $(\frac{1}{2}, \frac{1}{2})$

DOES ROTATING $A(1,0)$ BY 45° GIVE $(\frac{1}{2}, \frac{1}{2})$? NO! WHY?

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$$C = \cos \theta A + \sin \theta B$$



2D

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A. TRANSLATION

$$(1, 2) \text{ by } (3, 4) \rightarrow (4, 6)$$

B. ROTATION

$$(1, 2) \text{ by } 180^\circ \rightarrow (-1, -2)$$

C. SCALE

$$(1, 2) \text{ by } 3 \rightarrow (3, 6)$$

ROT: MATRIX MULT

$$\begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

SCALE: MATRIX MULT

$$\begin{pmatrix} S & 0 \\ 0 & S \end{pmatrix}$$

TRANS: NOT A MATRIX MULT

$$\underbrace{M_4 M_3 M_2 M_1}_M (P)$$

HOMOGENEOUS COORDS

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2D / CARTESIAN COORDS

$$P_c = (x, y)$$

$$P_H = (x_H, y_H, w)$$

WEIGHT

$$\left(\frac{x_H}{w}, \frac{y_H}{w} \right)$$

HOW $(1, 2, 3) \rightarrow \text{CART} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$
 $(2, 4, 6) \rightarrow \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$
 $\left(\frac{1}{3}, \frac{2}{3}, 1 \right) \rightarrow \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

YOU READ AHEAD FOR TUES
