CS 370 10/2/17

2D (x, y) → z = x + iy

$i^2 = -1$

(3, 4) → 3 + 4i

TRANSLATE BY (5, 6)

→ + 5 + 6i

(3 + 4i) + (5 + 6i) = 8 + 10i

ROT BY θ → × $e^{i\theta}$

θ = π/2 (90°) $e^{i\pi/2} = i$

(3, 4) (3 + 4i)i = −4 + 3i

(−4, 3)

QUATS

3 INDETERMINATES

i, j, k

$i^2 = -1 = j^2 = k^2$

$ij = k \quad ji = -k$

$jk = i \quad kj = -i$

$ki = j \quad ik = -j$

$q_1 = (1+2i) \quad q_2 = (3j+4k)$

$q_1 q_2 = 3j + 4k + 6ij + 8ik$

$ij = k, \quad ik = -j$

$= 3j + 4k + 6k - 8j$

$= -5j + 10k$

$q_2 q_1 = (3j+4k)(1+2i)$

$= 3j + 6ji + 4k + 8ki$

$ji = -k, \quad ki = j$

$= 3j - 6k + 4k - 8j$

$= 11j - 2k$

$$Q = a + bi + cj + dk$$

$$q_1 = (2 + 3k) \qquad q_2 = (3 - j)$$

$$q_1 q_2 = 6 \underbrace{i^2}_{-1} - 2 \underbrace{ij}_{k} + 9 \underbrace{ki}_{j} - 3 \underbrace{kj}_{-i}$$

$$= -6 - 2k + 9j + 3i$$

$$= \underline{-6 + 3i + 9j - 2k}$$

3D PT (x, y, z) $\quad p = (x + yj + zk)$

$(1, 2, 3) \qquad p = i + 2j + 3k$

TRANSLATION BY $(5, 6, 7) \rightarrow$ ADD $5 + 6j + 7k$

$(1, 2, 3)$ TRANS BY $(5, 6, 7)$

$$(i + 2j + 3k) + (5 + 6j + 7k)$$

$$= (6 + 8j + 10k) \qquad (6, 8, 10)$$

3) ROT θ ABOUT AXIS $\hat{a}$ NORMALIZED

$$\varphi = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\left(a_x i + a_y j + a_z k\right)$$

$$\rho' = \varphi\rho\varphi^*$$

ROT OF 180° ABOUT Z-AXIS $(0,0,1)$

$$\varphi = 0 + 1\left(0i + 0j + k\right)$$

$$\varphi = k$$

$$\rho' = \varphi\rho\varphi^* = k\rho(-k) = -k\rho k$$

$$\rho = (1,2,3) = i + 2j + 3k$$

$$\rho' = -k(i + 2j + 3k)k$$

$$= -(ki + 2kj + 3k^2)k$$

$$= -(kik + 2kjk + 3k^2)$$

$$= -(jk + 2(-i) - 3k)$$

$$= -(i + 2j - 3k) = -i - 2j + 3k$$

$$(-1,-2,3)$$

USE QUATS TO COMBINE 2 ROTS

1. 180° ABOUT Z AXIS $Q_1 = k$
2. 180° ABOUT Y AXIS $Q_2 = i$

$P' \; Q_2 Q_1 P Q_1^* Q_2^*$

COMBO ROT $Q = Q_2 Q_1$

$P' = Q P Q^*$

$Q = Q_2 Q_1 = ik = -j$

$P' = (-j) P (-j)^* = -jPj$

$Q = j \rightarrow 180°$ ABOUT Y AXIS.

ANIMATION

$$a = \left(\frac{3}{13}, \frac{4}{13}, \frac{12}{13}\right)$$

$\theta = 180°$

$$\varphi = \frac{3}{13} i + \frac{4}{13} j \rightarrow \frac{12}{13} k$$

ANIMATE IN 180 STEPS

$$Q_2 = 1^0 \qquad Q_2 = \cos\frac{10}{2} + \sin\frac{10}{2}\left(\frac{3}{13} i + \frac{4}{13} j + \frac{12}{13} k\right)$$