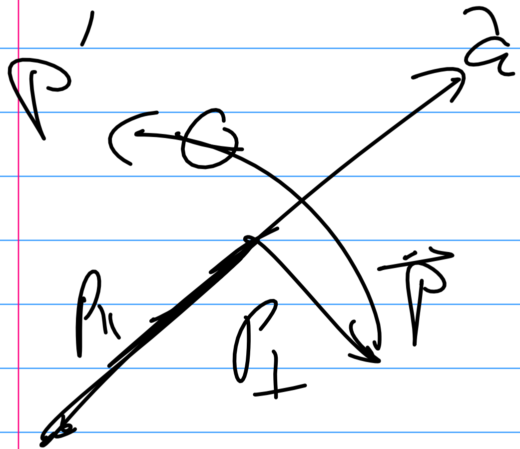
CG 9/26/17 p1




$a = (1, 2, 3)$

$|a| = 1^2 + 2^2 + 3^2$

$= 14$

$\vec{a} = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

$p = p_{\parallel} + p_{\perp}$

$p_{\parallel} = a \cdot p \, a$

$p_{\perp} = p - a \cdot p a$

Ex $\quad p = (1, 0, 0)$ $\qquad a \cdot p = \frac{1}{\sqrt{14}}$

$p_{\parallel} = \left(\frac{1}{14}, \frac{2}{14}, \frac{3}{14}\right)$

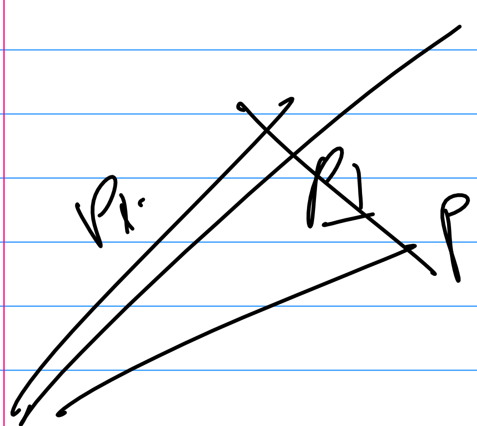
$$p = (1, 0, 0) \qquad \hat{a} = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$$

$$p_{\parallel} = \left(\frac{1}{14}, \frac{2}{14}, \frac{3}{14}\right)$$

$$\underline{p_{\perp}} = p - p_{\parallel} = \left(\frac{13}{14}, \frac{-2}{14}, \frac{-3}{14}\right)$$

$$\hat{a} \cdot p_{\perp} = \frac{1}{14\sqrt{14}}\left(13 - 4 - 9\right) = 0$$

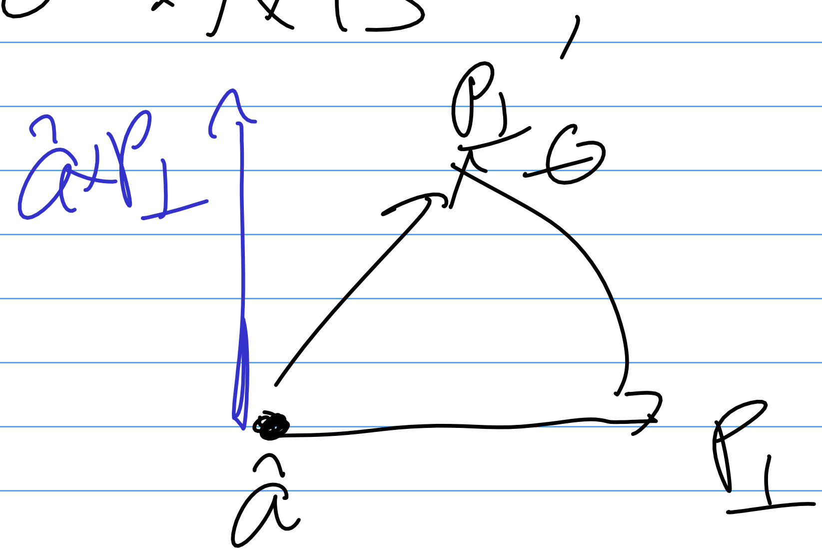
$-9$




POINTS ON AXIS DON'T MOVE

$$p_{\parallel}' = p_{\parallel}$$

$P_\perp$ ROTATES IN A PLANE $\perp$ TO AXIS,



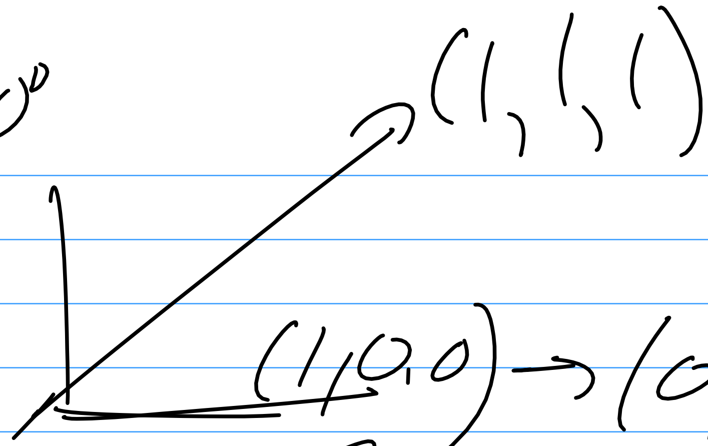

$$P_\perp' = \cos\theta\, P_\perp + \sin\theta\, \hat{a}\times P_\perp$$

$$p' = p_{\parallel} + \cos\theta\, P_\perp + \sin\theta\, \hat{a}\times p_\perp$$

$$= \hat{a}\cdot p\,\hat{a} + \cos\theta\left(p - \hat{a}\cdot p\,\hat{a}\right) + \underbrace{\sin\theta\left(\hat{a}\times\left(p - \cancel{\hat{a}\cdot p\,\hat{a}}\right)\right)}_{\sin\theta\,\hat{a}\times p}$$

$$\boxed{p' = \cos\theta\, p + (1-\cos\theta)\,\hat{a}\cdot p\,\hat{a} + \sin\theta\,\hat{a}\times p}$$

$\theta = 120^\circ$




$(1,0,0) \to (0,1,0) \to (0,0,1)$

$\hat{a} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \qquad p = (1,0,0)$

111
100
a-1

$\cos\theta = -\frac{1}{2} \qquad \sin\theta = \frac{\sqrt{3}}{2}$

$$p' = \underbrace{\underbrace{\cos\theta}_{-\frac{1}{2}}\, p}_{\left(-\frac{1}{2},0,0\right)} + \underbrace{\underbrace{(1-\cos\theta)}_{\frac{3}{2}}\, \underbrace{\underbrace{\hat{a}\cdot p}_{\frac{1}{\sqrt{3}}}\,\hat{a}}_{\left(\frac{1}{3},0,0\right)}}_{\left(\frac{1}{2},0,0\right)} + \underbrace{\underbrace{\sin\theta}_{\frac{\sqrt{3}}{2}}\, \underbrace{\hat{a}\times p}_{\frac{1}{\sqrt{3}}(0,1,-1)}}_{\left(0,\frac{1}{2},-\frac{1}{2}\right)}$$

$$= \left(0, \frac{1}{2}, -\frac{1}{2}\right)$$

?!! X

WANT TO GET A MATRIX $M$ FROM $\hat{a}, \theta$

$$\boxed{p' = Mp}$$

$c = \cos\theta$

1/ $p_1' = \cos\theta\, p$ $\quad M_1 = \begin{pmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{pmatrix}$

$p_1' = M_1 p$

2/ $p_2' = (1-\cos\theta)\, \hat{a} \cdot p\, \hat{a}$

(WANT A MATRIX $Q$. $Q$ DEPENDS ON $\hat{a}$.

$Qp = \hat{a} \cdot p\, \hat{a}$

$$Q = \begin{pmatrix} a_1^2 & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_3^2 \end{pmatrix}$$

$$a = (7, 7, 0)$$

$$Q = \begin{array}{c} \\ \\ \\ \end{array}\begin{pmatrix} 49 & 49 & 0 \\ 49 & 49 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{array}{c} 0 \\ 1 \\ 2 \end{array}\begin{pmatrix} 3 \\ 5 \\ 9 \end{pmatrix} = \begin{pmatrix} 392 \\ 392 \\ 0 \end{pmatrix}$$

$$Qp = a \cdot pa$$

$$p = (3, 5, 9)$$

$$a \cdot p = 56 \qquad a \cdot pa = (56 \cdot 7, 56 \cdot 7, 0)$$

$$(392, 392, 0)$$

$$p = (0, 1, 2)$$

$$Qp = (49, 49, 0) \qquad a = (7\ 7\ 0)$$

$$a \cdot p = 7 \qquad a \cdot pa = (49, 49, 0)$$

$$M_2 = (1 - \cos\theta)\left[a_i a_j\right]$$

$$a \times p = Rp$$

$$M_3 = \sin\theta\, R$$

$$M = M_1 + M_2 + M_3$$

$$R = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}$$

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$$M \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} m_{11} \\ m_{21} \\ m_{31} \end{pmatrix} = C_1$$

$$|C_1| = 1 \qquad C_1 \cdot C_1 = 1 = C_2 \cdot C_2 = C_3 \cdot C_3$$

$$C_1 \cdot C_2 = 0 = C_1 \cdot C_3 = C_2 \cdot C_3$$

$$C_i \cdot C_j = \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

$|M|$ IS HOW A VOLUME SCALES

$|M| = 1$

$$M_{11} + M_{22} + M_{33}$$
$$= 3\cos\theta + (1-\cos\theta)\underbrace{(a_1^2 + a_2^2 + a_3^2)}_{1}$$
$$= 1 + 2\cos\theta$$

# 2D ROTATION

$(x, y) \longleftrightarrow x + iy$

COMPLEX #

2D PT

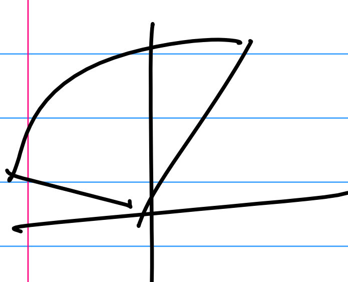
$(3, 4)$ $\qquad 3 + 4i$

ROTATE $\theta$ $\qquad \times e^{i\theta}$

$\theta = 180^\circ, \pi$ $\qquad e^{i\pi} = -1$


$(3 + 4i)(-1) = (-3 - 4i)$

$90^\circ = \frac{\pi}{2}$ $\qquad e^{i\frac{\pi}{2}} = i$



$(3 + 4i)(i)$

$= -4 + 3i$