

3D CARTESIAN

(x, y, z) e.g. $(1, 2, 3)$

HOMOGENEOUS (HC)

(x, y, z, w)

e.g. $(1, 2, 3, 4)$ WEIGHT = $(\frac{1}{4}, \frac{2}{4}, \frac{3}{4})$

HC $(x, y, z, w) \rightarrow \text{COORD} \left(\frac{x}{w}, \frac{y}{w}, \frac{z}{w} \right)$

CART

$(5, 6, 7)$

HC

$(5, 6, 7, 1)$

$(50, 60, 70, 10)$

$(10, 12, 14, 2)$

$$A \otimes B = (D_x, D_y, D_z)$$

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$$= \text{MULT HC BY} \begin{pmatrix} 1 & 0 & 0 & D_x \\ 0 & 1 & 0 & D_y \\ 0 & 0 & 1 & D_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A \otimes B = (1, 2, 3)$$

$$= \begin{pmatrix} 3 \\ 3 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 2 \\ 1 \end{pmatrix}$$

Q) TEST ON (2, 1, 2) → (2, 1, 2, 1)

(CASE (3, 3, 5) → (4, 2, 4, 2))

$$(3, 3, 5) \begin{pmatrix} 6 \\ 6 \\ 10 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 4 \\ 2 \end{pmatrix}$$

SCALE BY (s_1, s_2, s_3)

MULT BY

$$\begin{pmatrix} s_1 & 0 & 0 & 0 \\ 0 & s_2 & 0 & 0 \\ 0 & 0 & s_3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

SCALE BY $(2, 3, 4)$

(406)

$(8, 0, 24)$

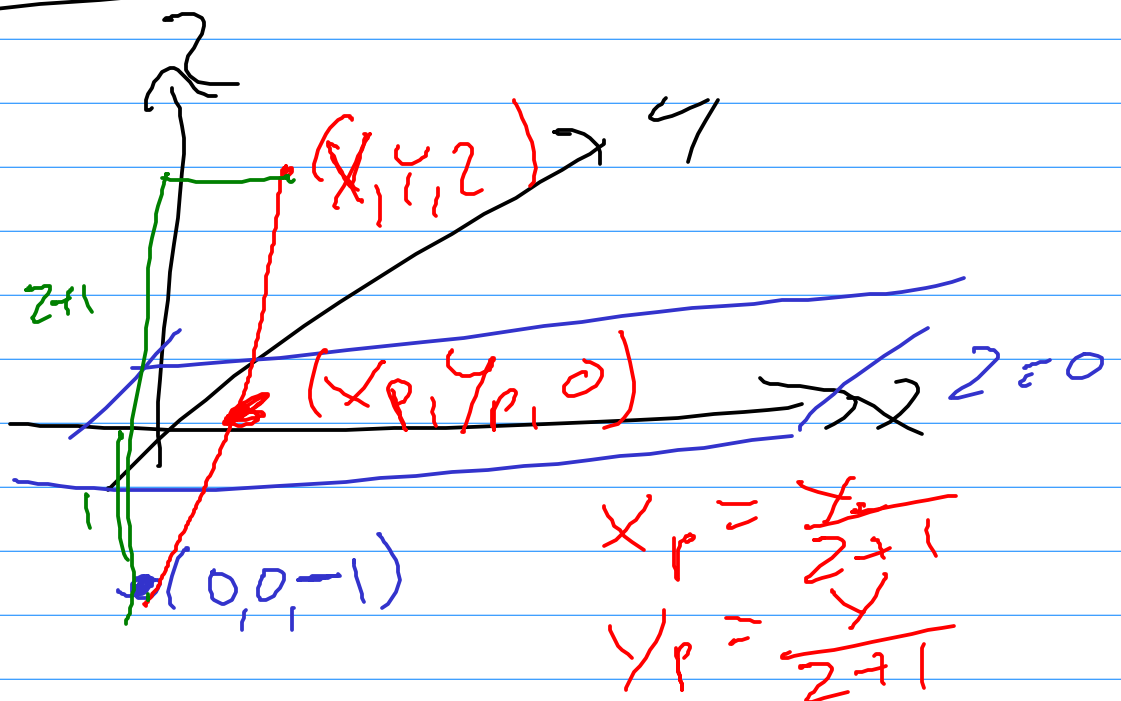
$$\begin{pmatrix} 8 \\ 0 \\ 24 \\ 1 \end{pmatrix} = \left(\begin{array}{cccc|c} 2 & 0 & 0 & 0 & 4 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 6 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right)$$

ROT 3+3

$$\begin{pmatrix} m_{11} & m_{12} \\ & & & \\ & & & \\ & & & & m_{33} \end{pmatrix}$$

$$HC \begin{pmatrix} \boxed{3 \times 3} & 0 \\ & 0 \\ & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

PROJECTION



IF ORIGINAL POINT HAS $z=0$
PROJECTED POINT IS SAME

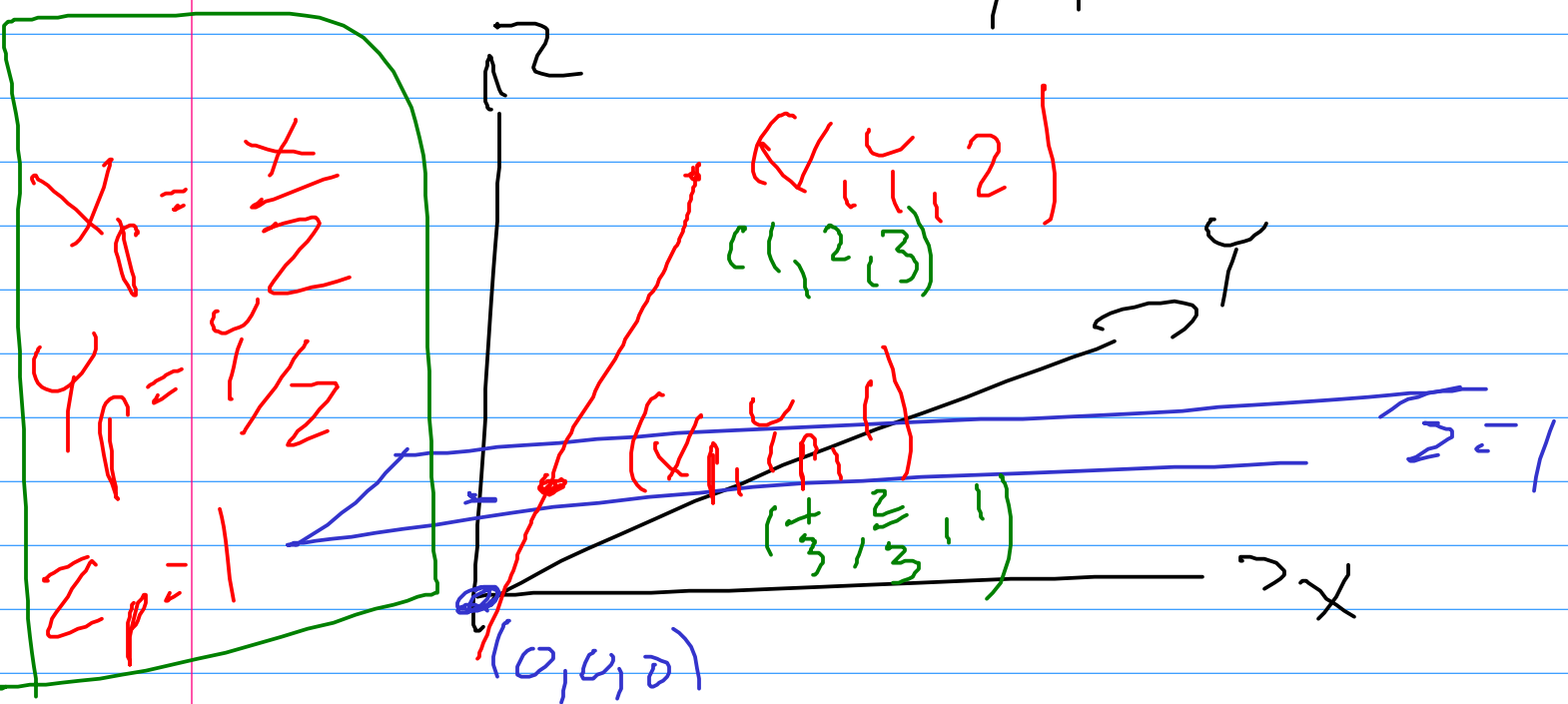
$$x_p = \frac{x}{0+1} = x \quad \checkmark$$

$$y_p = y \quad \checkmark$$

ANOTHER CASE

PROJECTION PLANE $z=1$

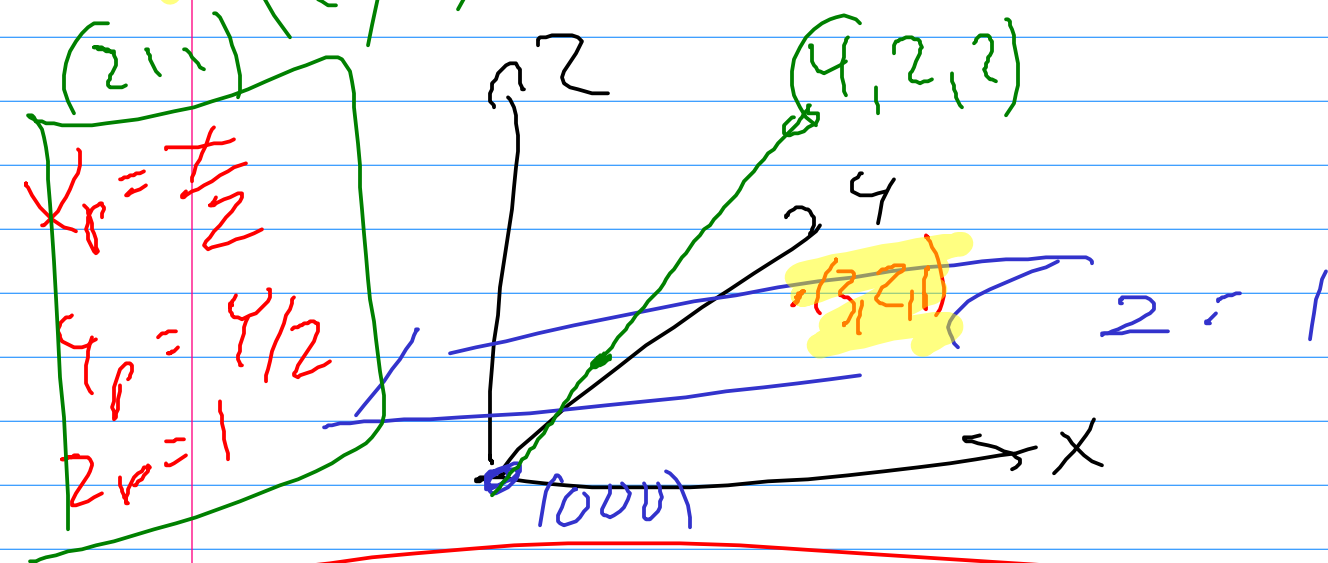
CENTER $(0,0,0)$



$$x_p = \frac{x}{2} \quad y_p = \frac{y}{2} \quad z_p = 1 \quad G$$

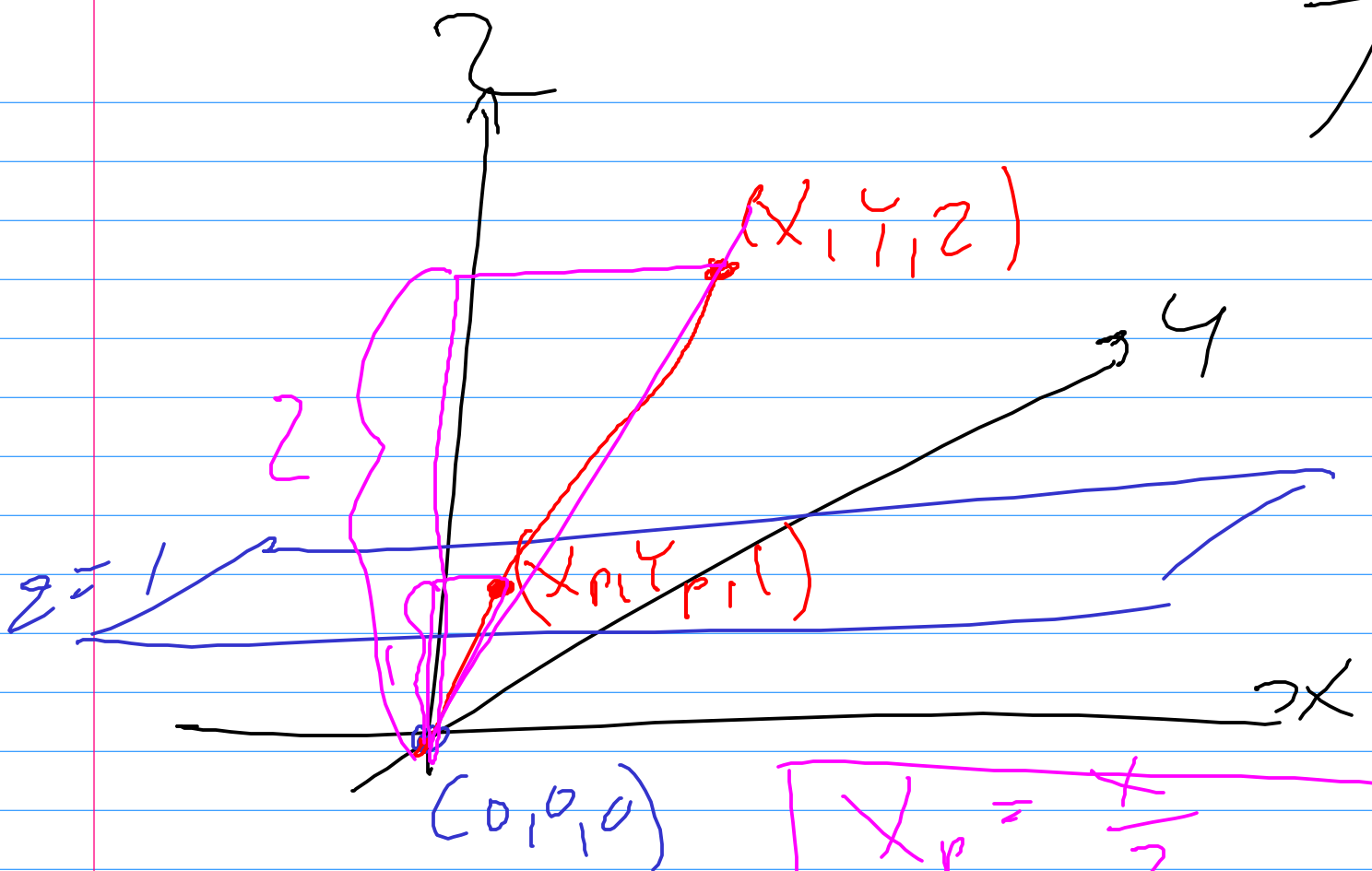
IS ALSO A XY MATRIK

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 2 \\ 2 \end{pmatrix} \begin{pmatrix} 15 \\ 10 \\ 5 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 15 \\ 10 \\ 5 \\ 5 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 2 \\ 1 \end{pmatrix}$$



~~part~~ HC $(x, y, z, w) \rightarrow (\frac{x}{w}, \frac{y}{w}, \frac{z}{w}, 1)$

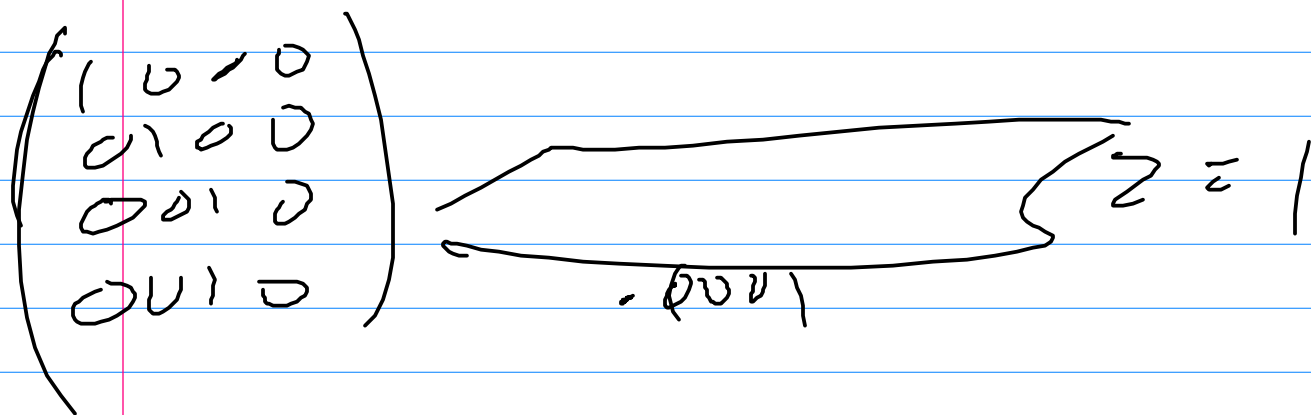
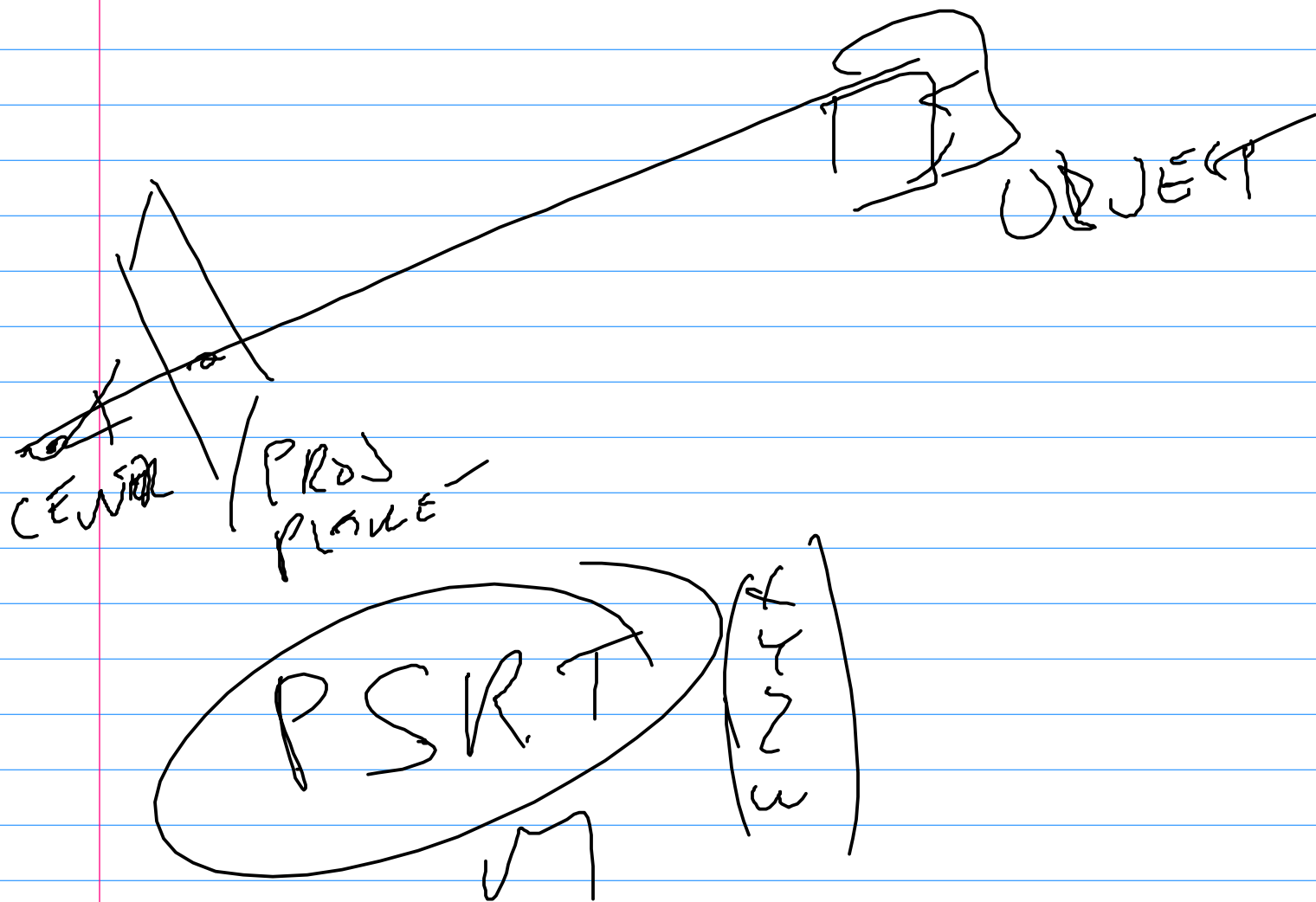
$\frac{4}{5} = 2$
 $\frac{4}{5} = 3$
 $(15, 10, 5, 5) \rightarrow (3, 2, 1)$

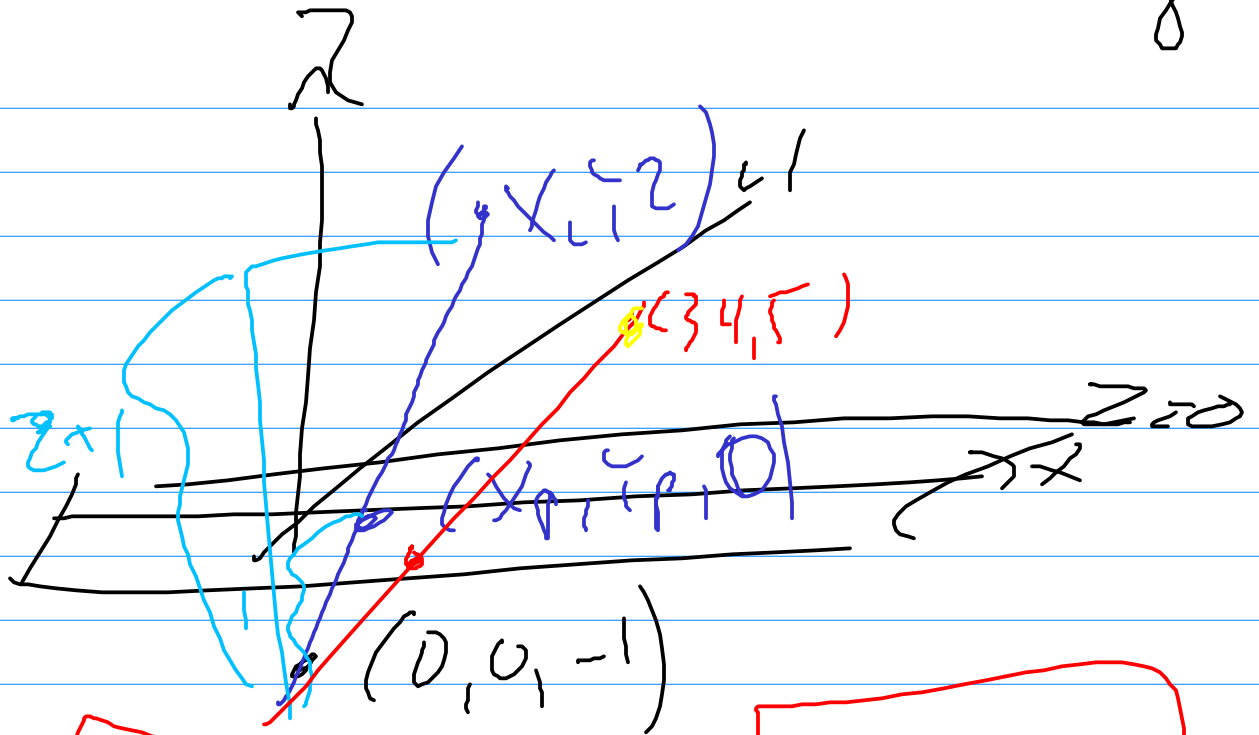


$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \\ 1 \end{pmatrix}$$

HOW IS THIS USEFUL?





$$x_0 = \frac{x}{z+1}$$

$$y_0 = \frac{y}{z+1}$$

$$(3, 4, 5) \rightarrow$$

$$\left(\frac{1}{3}, \frac{2}{3}, 0\right)$$

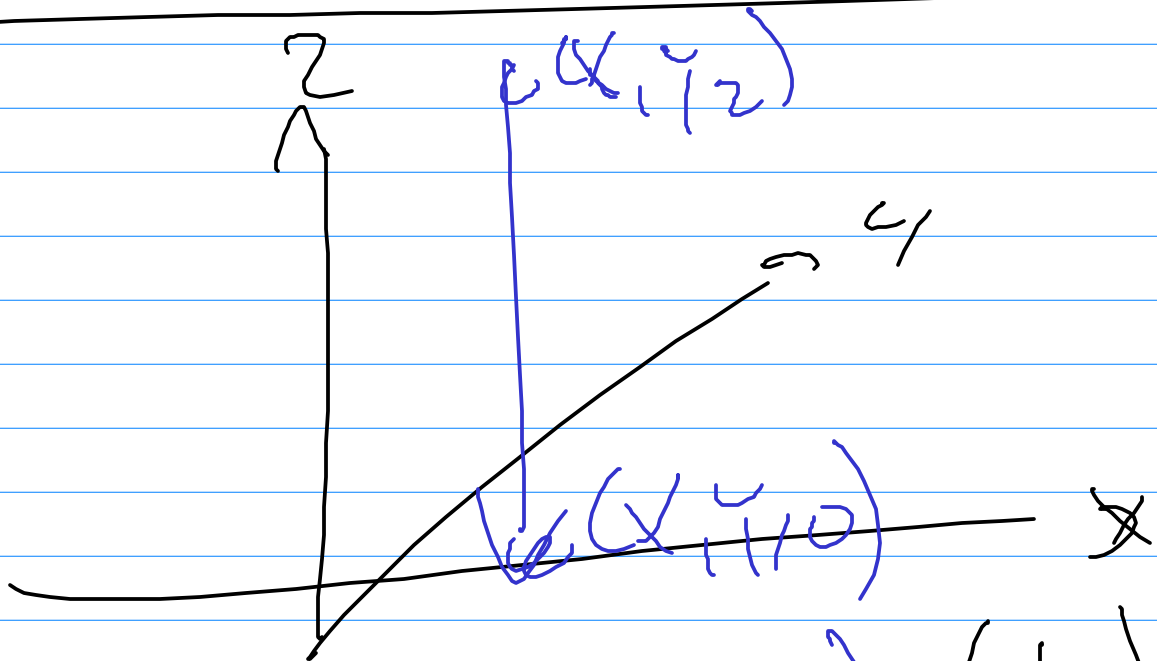
$$z_0 = 0$$

$$\begin{pmatrix} 1/2 \\ 2/3 \\ 0 \end{pmatrix}$$

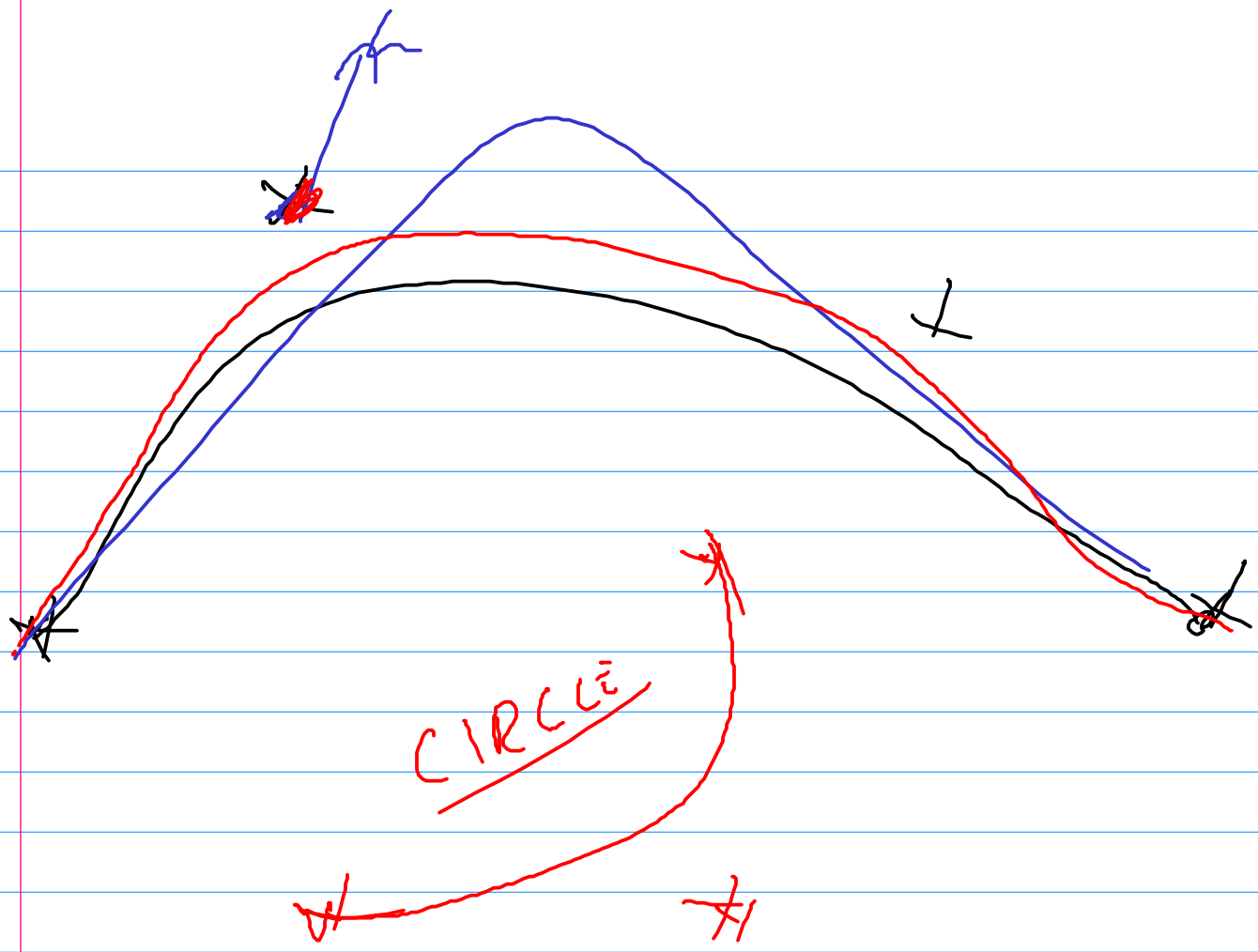
$$\begin{pmatrix} 9 \\ 12 \\ 0 \\ 18 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 9 \\ 12 \\ 15 \\ 3 \end{pmatrix}$$

PARALLEL PROJECTION



$$\begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}$$



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