CS 9/28/16 p1

GOAL: SPEC ROTATION WITH { AXIS, ANGLE }

WANT TO COMBINE ROTATIONS + FIND THE COMBO'S AXIS ANGLE

HOW? EXTEND COMPLEX NUMBERS

$z = x + iy \qquad i^2 = -1$

POINT $(3,4) \Rightarrow z = 3 + 4i$

TO ROTATE BY θ

MULTIPLY BY $e^{i\theta}$

WHAT THAT MEANS IS DONE IN AN EARLIER MATH COURSE.

ANGLE IN RADIANS

e.g. $90^\circ \to \frac{\pi}{2}$ RADIANS

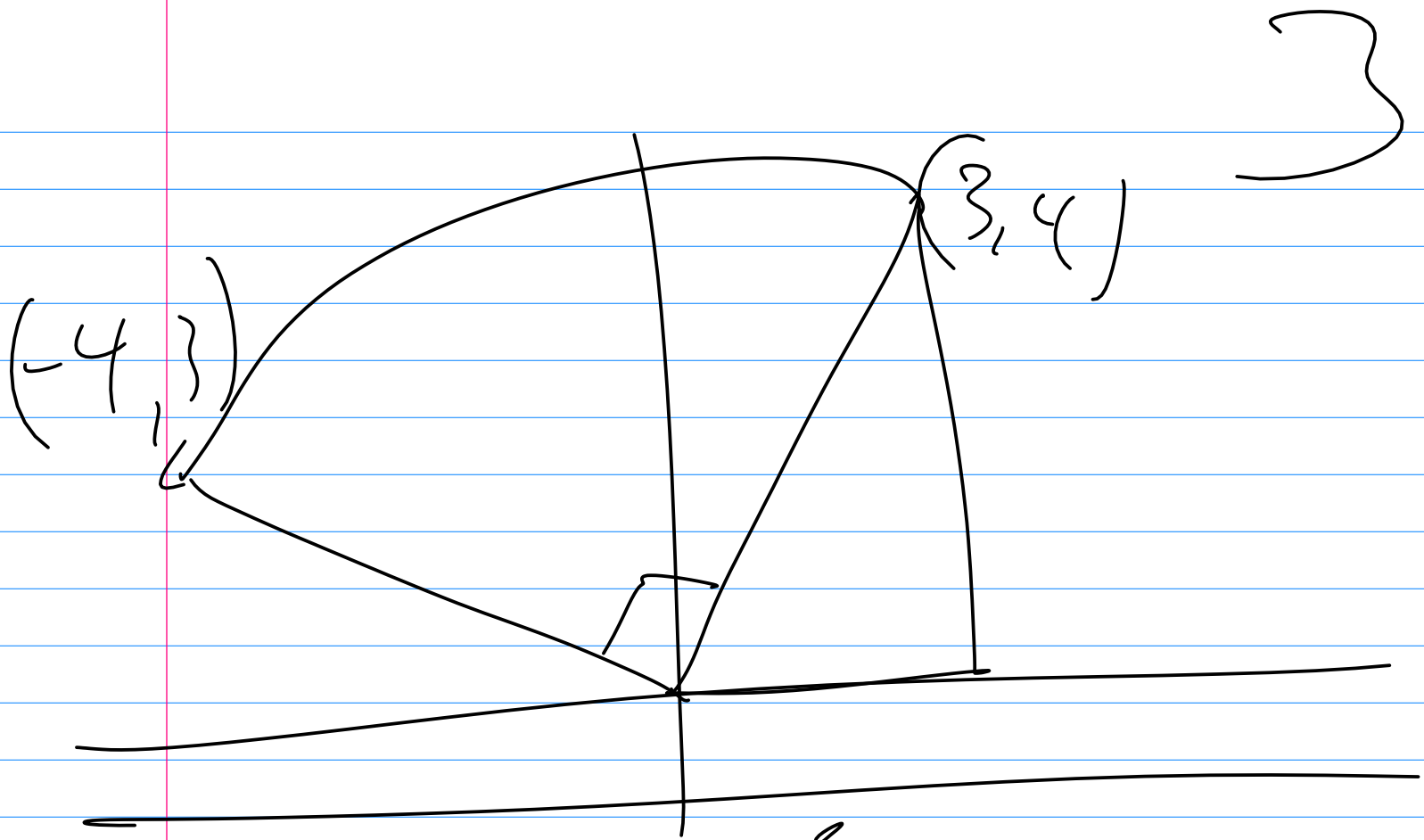
ROTATE BY 90° $\xrightarrow{\text{MULTIPLY BY } e^{i\frac{\pi}{2}} = i}$

ROTATE $(3,4)$ $\quad 3+4i$

BY 90° $\quad 2: (3+4i)\,i = 3i + 4i^2$

$= -4+3i$

$(-4,3)$



QUATERNIONS

HAVE $1, i, j, k$

RULES $i^2 = j^2 = k^2 = -1$

$ij = k$ ETC $ji = -k$

$q = A + Bi + Cj + Dk$

$(1+2j)(3k) = 3ik + 2jk$

$= -3j + 2i$

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3D POINT (x, y, z)

→ QUAT $p = xi + yj + zk$

ROTATION BY θ ABOUT AXIS (A_x, A_y, A_z)

$$Q = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}(A_x i + A_y j + A_z k)$$

$$p' = QpQ^*$$

FOR THE ROTATION WE WANT TO DO, COMPUTE A Q.

THEN USE Q TO DO ROTATION.

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I WANT TO COMBINE 2 ROTATIONS

1st 180° ABOUT X AXIS

2nd 180° ABOUT Y AXIS

WHAT IS RESULT?

$$Q_1 = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}(A_x i + A_y j + A_z k)$$

$\theta = 180°$ $\frac{\theta}{2} = 90°$ $\cos\frac{\theta}{2} = 0$ $\sin\frac{\theta}{2} = 1$

X AXIS: $(1, 0, 0)$

$Q_1 = 0 + 1(1i + 0j + 0k) = i$

$Q_1^* = -i$

TO ROTATE $P' = Q P Q^*$

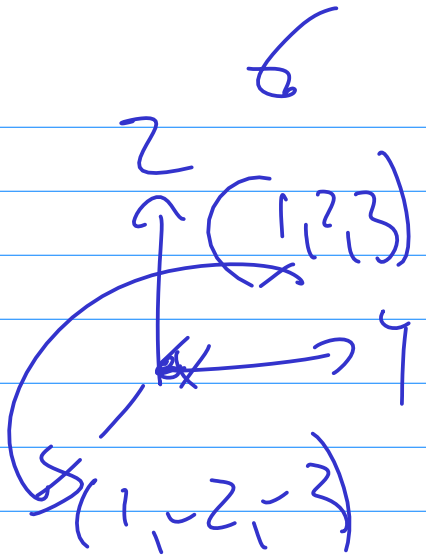
$P = (1, 2, 3)$ $P = 1 + 2j + 3k$

$P' = -1(1 + 2j + 3k)1$

$= -(-1 + 2k - 3j)i$

$= (-i + 2j + 3k)$

$= (-1, 2, 3)$



180° ABOUT X AXIS $Q_1 = i$

2: 180° ABOUT Y AXIS

$Q_2 = j$

ROTATE $(1, 2, 3)$ BY

$j(i + 2j + 3k)(-j)$

$= (-k - 2 + 3i)(-j)$

$= (-i + 2j - 3k)$ $(-1, 2, -3)$

$Q_1 = i$

$Q_2 = j$

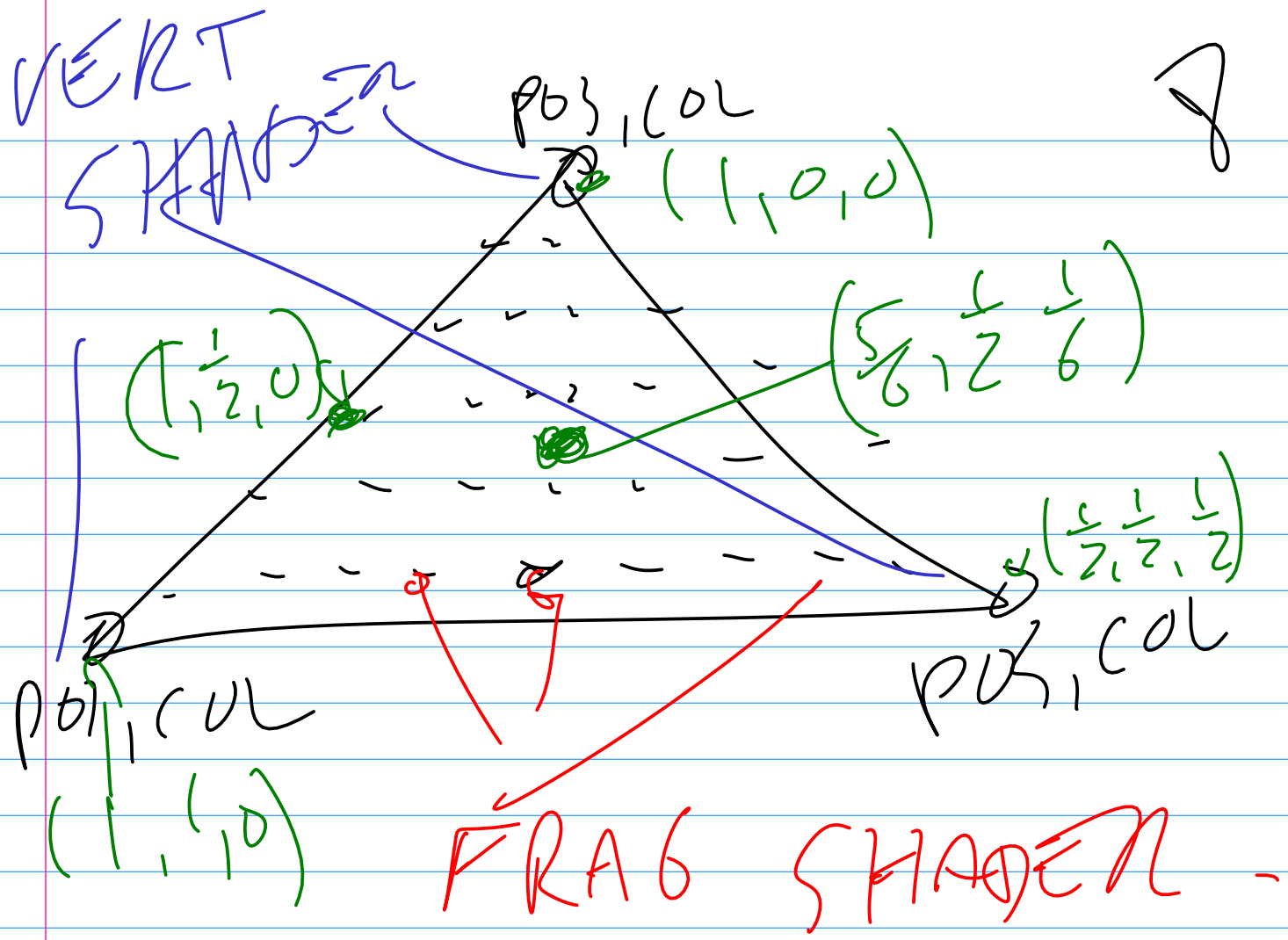
THEN $Q = Q_2 Q_1$

$= ji$

$= -k$

$-k = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}(0i + 0 - k)$

$\theta = 180^\circ$ ABOUT -Z AXIS


VERT SHADER
POS, COL
(1,0,0)
(1/2, 0)
(1/2, 1/2, 0)
(1/2, 1/2, 1/2)
POS, COL
POS, COL
(1, 0, 0)
FRAG SHADER