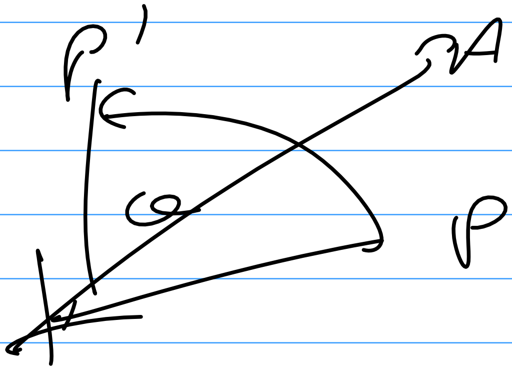
CG 9/19/16 p1

3D ROTATION

Q USER SPEC?

A AXIS + ANGLE



$|A| = 1$

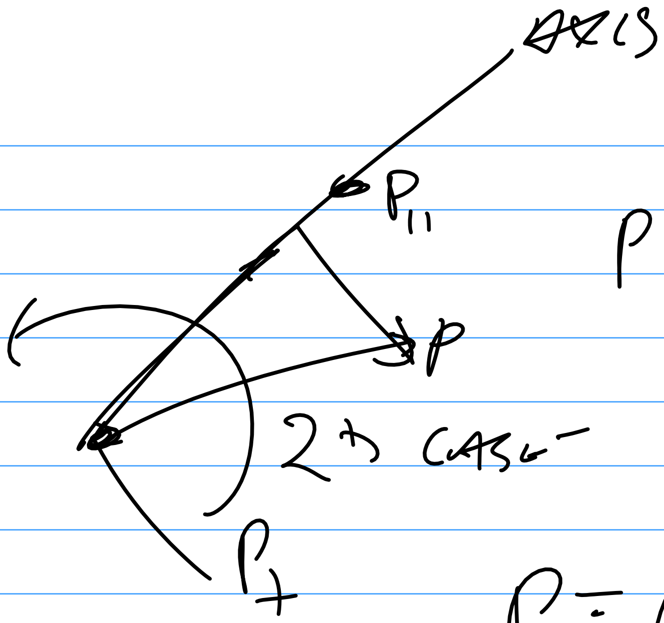
$P' = MP$ $\quad M = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

3D ABOUT Z AXIS

REALLY 2D

Z STAYS SAME

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



$P \parallel A \rightarrow P' = P$

$$P = P_\parallel + P_\perp$$

$P_\parallel = A \cdot P\, A$ $P_\perp = P - P_\parallel$

WANT MATRIX TO ROTATE P

$$P' = M P$$

DEPENDS ON A, Θ

GIVEN A MATRIX IS IT A ROTATION?

$$\begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & & \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} m_{11} \\ m_{21} \\ m_{31} \end{pmatrix}$$

LEN=1

3

REFLECTION

$$M = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

IF λ IS EIGENVALUE FOR EIGENVECTOR v OF M

THEN $Mv = \lambda v$

2D

POINT $(x,y) \Leftrightarrow$ COMPLEX $x+iy$

$(3,4) = \quad z = 3+4i$

TRANSLATE BY 5 OVER, 6 UP

ADD $(5+6i) \longrightarrow (8+10i)$

ROTATE BY θ =

MULTIPLY BY $e^{i\theta}$

90° = $\frac{\pi}{2}$ RADIANS $\quad e^{i\frac{\pi}{2}} = i$

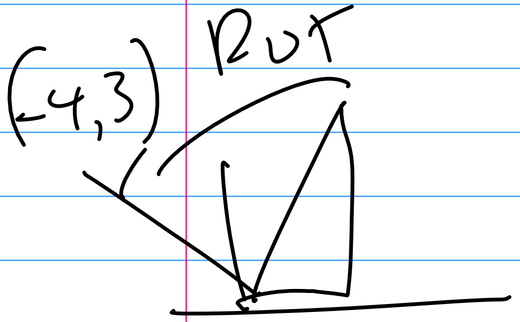
ROT BY 90° = MULTIPLY BY i

$(3,4) \qquad 3+4i$

$(3+4i)\,i = 3i+4i^2$

$= -4+3i$

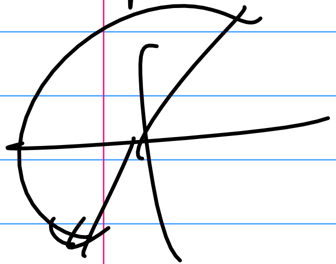
PT $(-4,3)$



$180^\circ \quad \theta = \pi$

$$e^{i\pi} = -1$$

ROT $(3,4)$ $\quad 3+4i \rightarrow -3-4i$



$(-3,-4)$

WHAT IS i ?

WE DON'T KNOW WHAT IT IS, BUT WE KNOW HOW TO WORK WITH IT.

ADD $2i+3i = 5i$

ADD TO REAL $2+3i$

MULT: $i^2 = -1$

$$(1+2i)(3+4i) = 3+4i+6i+\cancel{8i^2}$$

-8

$-5+10i$

QUATERNIONS

LIKE COMPLEX BUT MORE

HAVE i, j, k

RULES $i^2 = -1$ $j^2 = -1$ $k^2 = -1$

$ij = k$ $ji = -k$

$jk = i$ $kj = -i$

$ki = j$ $ik = -j$

NOT COMMUTATIVE

3D ROT NOT COMMUT.

ROTATE STOOL

R_1 : 90° ABOUT VERTICAL AXIS

R_2 - 90° ABOUT AXIS FRONT-BACK IN ROOM

$R_1 R_2 \neq R_2 R_1$.

$P(X, Y, Z) = Xi + Yj + Zk$

$(1, 2, 3) \rightarrow i + 2j + 3k$

TRANSLATE BY ADDING

FOR ROTATION BY θ ABOUT A $|A| = 1$

DEFINE $Q = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}(A_x i + A_y j + A_z k)$

$90°$ ABOUT Z AXIS $A = (0, 0, 1)$

$\cos\frac{\theta}{2} = .7 = \sin\frac{\theta}{2}$

$Q = .7 + .7K$ $Q^* = \sin\frac{\theta}{2} - \cos\frac{\theta}{2}(\sim)$

DEFINE $Q^* = .7 - .7K$

$P' = Q P Q^*$

ROT $(1, 2, 3)$ BY $90°$ ABOUT Z AXIS

$P' = (.7 + .7K)(i + 2j + 3k)(.7 - .7k)$

$= (.7i + 1.4j + 2.1K + .7j - 1.4i - 2.1)(.7 - .9k)$

$= (-2.1 - .7i + 2.1j + 2.1K)(.7 - .7K)$

$= .5(-3 - i + 3j + 3k)(1 - k)$

$$p' = .5(-3 - i + 3j + 3k)(1 - k)$$

$$= .5(-3 - i + 3j + 3k + 3k - j + 3i + 3)$$

Correction: -3i

$$= .5(0 \overset{-4i}{\cancel{+2i}} + 2j + 6k)$$

$$= (-2\, i + j + 3k) \rightarrow (\underset{-2}{\cancel{1}}, 1, 3)$$

$$\Rightarrow\Leftarrow$$

WHAT WE WANTED   ROT (1,2,3) BY 90° ABOUT (0[illegible]

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$(-2, 1, 3)$$

IF YOU CAN FIGURE WHAT WENT WRONG YOU'LL UNDERSTAND THIS.