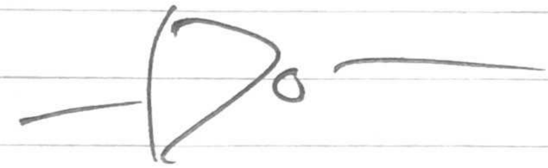
C2 9/3/20 - 1

BIT: 0, 1



QBIT

$q = \begin{pmatrix} a \\ b \end{pmatrix}$ ← complex $|a|^2 + |b|^2 = 1$

$a|0\rangle + b|1\rangle$

$q_1 = \begin{pmatrix} a \\ b \end{pmatrix}$ $q_2 = \begin{pmatrix} c \\ d \end{pmatrix}$

$q_1 q_2 = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}$

Mq

HADAMARD GATE

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{a+b}{\sqrt{2}} \\ \frac{a-b}{\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\sqrt{NOT}$ $\quad \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}$

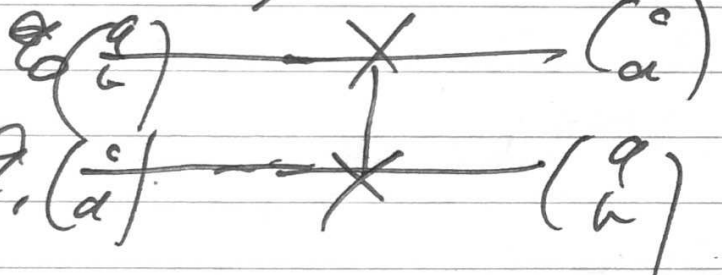
$\sqrt{NOT}\ \sqrt{NOT} = NOT$

PHASE SHIFT $\begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$

---

2 QBIT GATES

SWAP



$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}^T \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}$

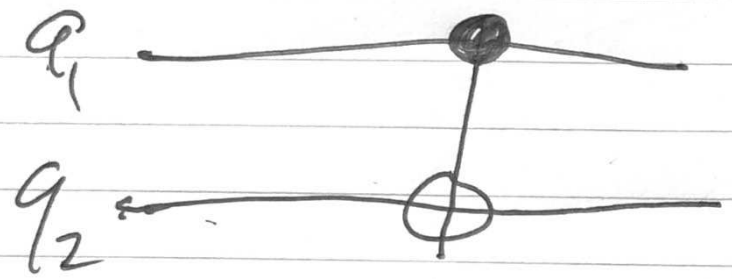
$\longrightarrow$ TIME

GATE: 4x4 MATRIX

$$\begin{pmatrix} 1&0&0&0 \\ 0&0&1&0 \\ 0&1&0&0 \\ 0&0&0&1 \end{pmatrix} \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix} = \begin{pmatrix} ac \\ bc \\ ad \\ bd \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix}^T \begin{pmatrix} a \\ b \end{pmatrix}$$

$\sqrt{\text{SWAP}}$

CNOT CONTROLLED NOT



$q_1 = 1 \Rightarrow$ INVERT Q_2

$$\begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{pmatrix} \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bd \\ bc \end{pmatrix} =$$

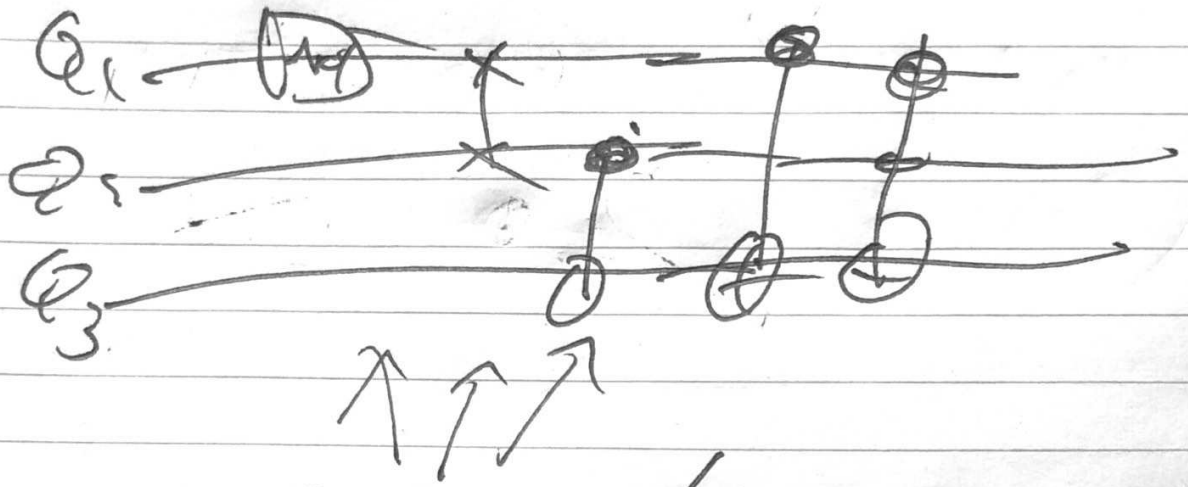
THIS GATE IS MORE SOPHISTICATED THAN APPARENT

$q_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ OR $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ IS EASY.

$q_1 = \begin{pmatrix} a \\ b \end{pmatrix}$ IS TRICKY.

RESULT OFTEN CANNOT BE DECOMPOSED INTO PRODUCT OF 2 SEPARATE QBITS.

ENTANGLEMENT



OPERATOR/MATRIX

TIME →

STATE = $\begin{pmatrix} A_1 \\ \downarrow \\ A_7 \end{pmatrix}$ $\overset{8\times 8}{\rightarrow}$ $\begin{pmatrix} A_0' \\ \downarrow \\ A_7' \end{pmatrix}$