EXACT INTERSECTION OF 3D GEOMETRIC MODELS

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EXACT INTERSECTION OF 3D GEOMETRIC MODELS

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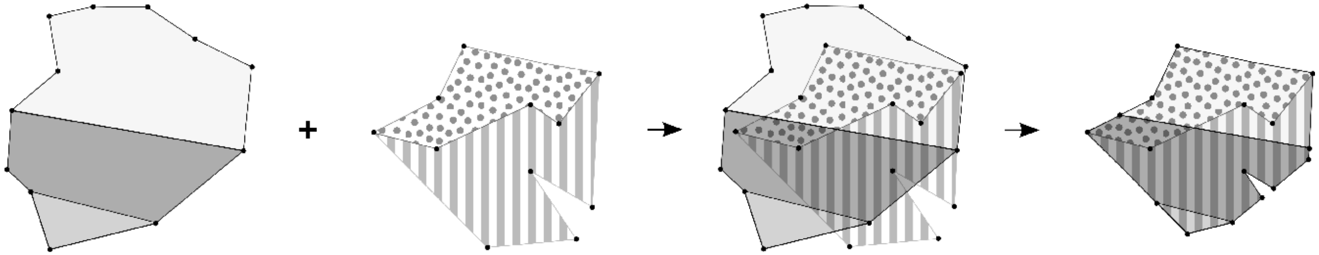
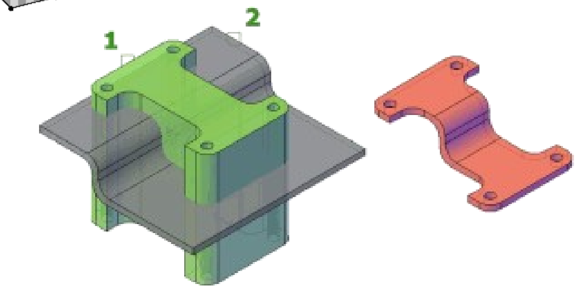
Mesh intersection

- Polygonal map overlay/intersection: important GIS problem

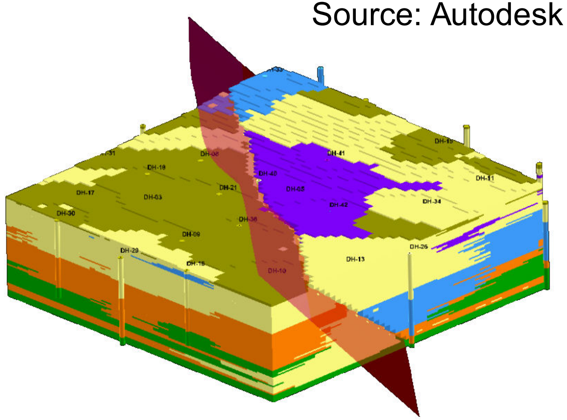


- 2D intersection also extends to 3D.



Source: Autodesk

- Examples:
  - CAD: intersection of industrial parts.
  - GIS: terrain models (layers of soil in a mine, volume that will be dug, city models, etc)

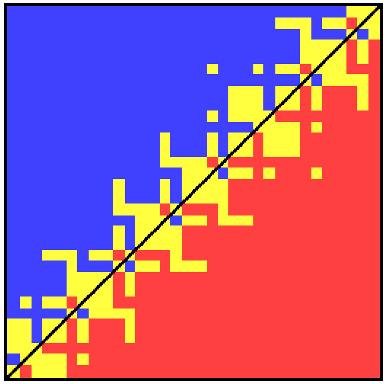


Source: Rockworks

- Our focus: 3D triangulated meshes

Challenge

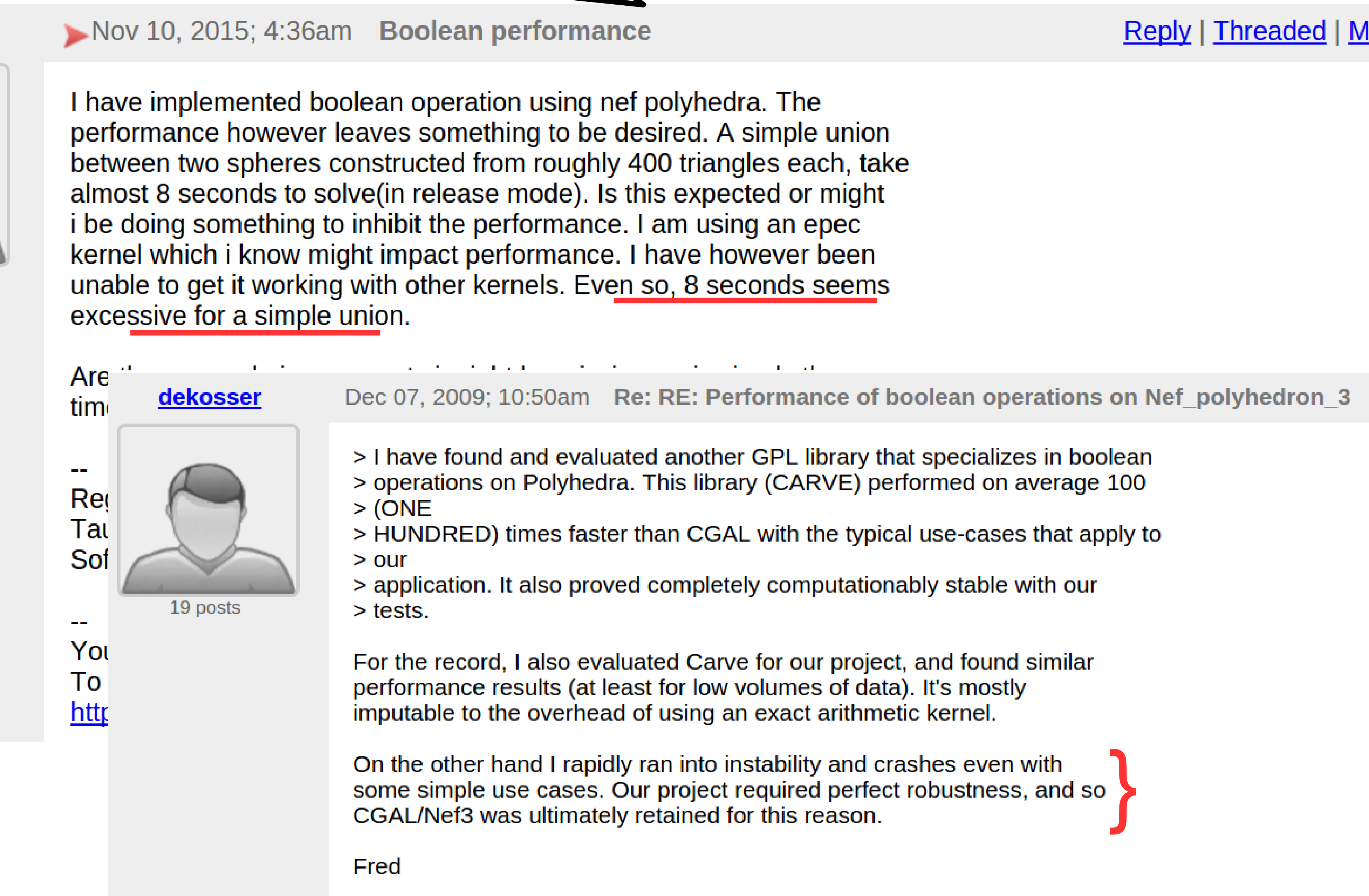
- Finite precision of floating point →roundoff errors.
  - Common techniques (snap rounding, epsilon tweaking, etc): no guarantee.



Source: Kettner et al., Classroom examples of robustness problems in geometric computations

- Big amount of data & 3D→ increase problem.
- Exactness and performance: very important (e.g. guaranteed subroutine)

Examples from CGAL mailing list (there are several other similar threads): People want exactness and performance!



Nov 10, 2015; 4:36am Boolean performance

I have implemented boolean operation using nef polyhedra. The performance however leaves something to be desired. A simple union between two spheres constructed from roughly 400 triangles each, take almost 8 seconds to solve(in release mode). Is this expected or might i be doing something to inhibit the performance. I am using an epec kernel which i know might impact performance. I have however been unable to get it working with other kernels. Even so, 8 seconds seems excessive for a simple union.

Are
tim

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dekosser

Dec 07, 2009; 10:50am Re: RE: Performance of boolean operations on Nef_polyhedron_3

> I have found and evaluated another GPL library that specializes in boolean
> operations on Polyhedra. This library (CARVE) performed on average 100
> (ONE
> HUNDRED) times faster than CGAL with the typical use-cases that apply to
> our
> application. It also proved completely computationably stable with our
> tests.

For the record, I also evaluated Carve for our project, and found similar performance results (at least for low volumes of data). It's mostly imputable to the overhead of using an exact arithmetic kernel.

On the other hand I rapidly ran into instability and crashes even with some simple use cases. Our project required perfect robustness, and so CGAL/Nef3 was ultimately retained for this reason.

Fred

Key techniques

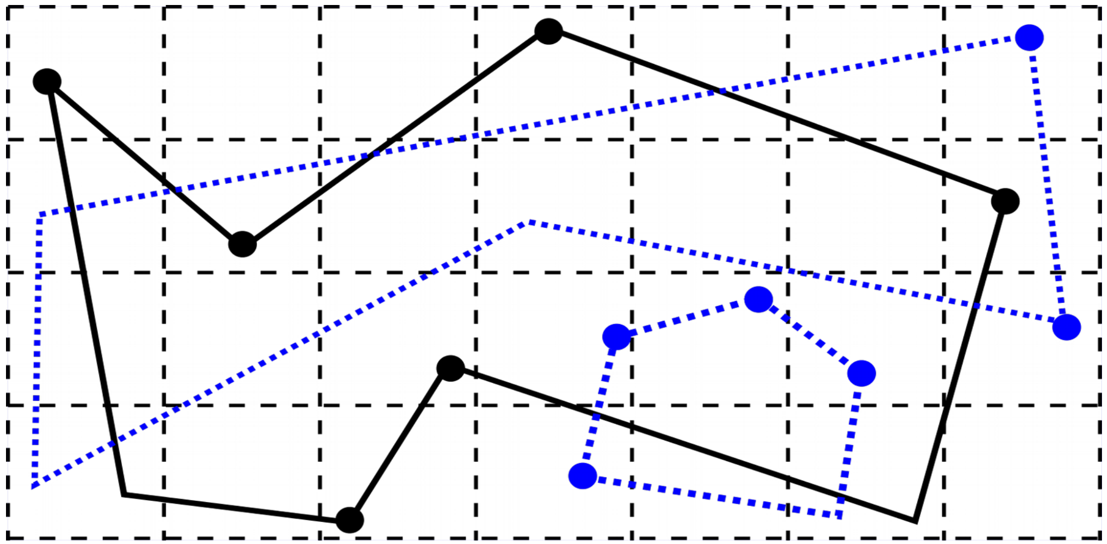
- We've been using a combination of 5 techniques
  - Arbitrary precision rational numbers: for exactness.
  - **Simulation of Simplicity**: for ensuring all the special cases are properly handled.
  - Simple data representation and local information: parallelization and correctness.
  - Parallel programming: explore better the computing capability of current hardware.
  - **Two-level uniform grid**: accelerate computation; quickly constructed in parallel.

Example: computing intersections

- “Brute force”: O(|A| x |B|)
- Other possible techniques:
  - Sweep-line
  - Complicate and doesn't parallelize
- Uniform grid
  - Tests: very efficient

Uniform grid

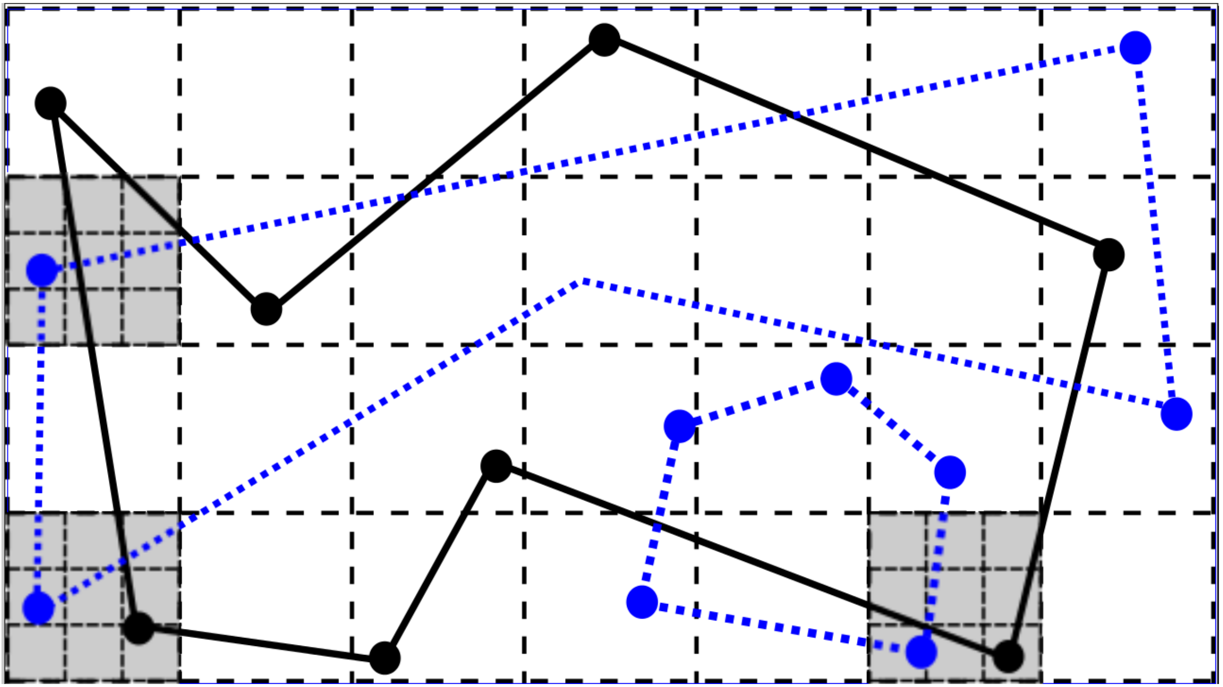
- Insert edges in grid cells (edge may be in several cells).
- For each grid cell *c*, compute intersections in *c*.
  - 3D version is analogous



4x7 uniform grid.
Blue map: 8 edges
Black map: 16 edges

Uniform grid

- Uniform Grids work well for uneven data.
- For very uneven data: 2-level uniform grid.



Simulation of Simplicity

- Special/degenerate cases
  - Usually difficult to handle
  - Mainly in 3D

- How to handle them efficiently and effectively?

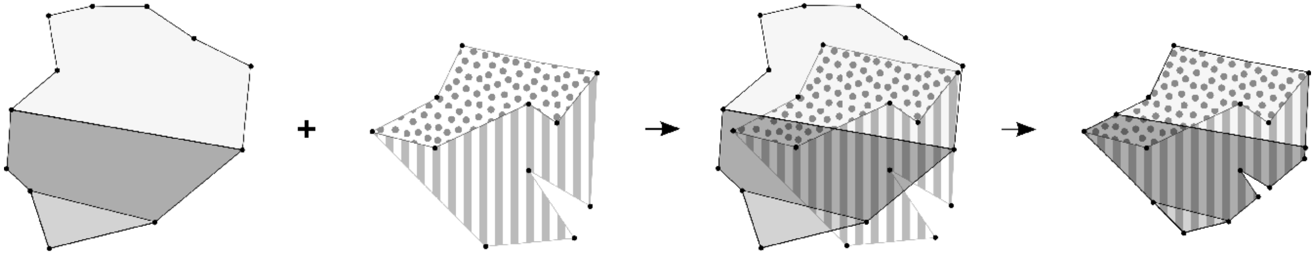
- Simulation of Simplicity (SoS), Edelsbrunner and Mücke:
  - Simple and efficient general purpose technique.
  - Globally consistent
  - Basic idea: if points are perturbed, the degeneracies in geometrical problems will disappear and do not need to be treated.

Simulation of Simplicity

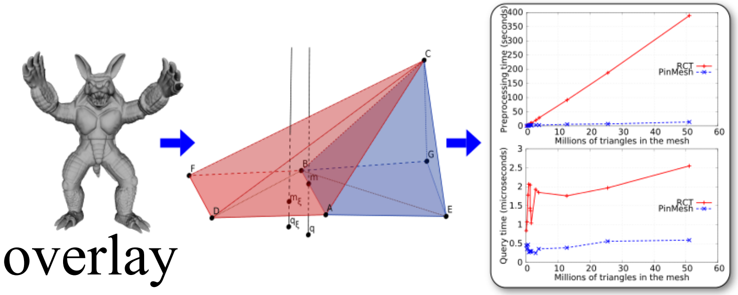
- Perturbation
  - Points are perturbed using orders of infinitesimals ε^i
  - Infinitesimal: indeterminate (code simulates the *effect* of the infinitesimals – we do not actually use specific infinitesimals).

Our previous works using these techniques

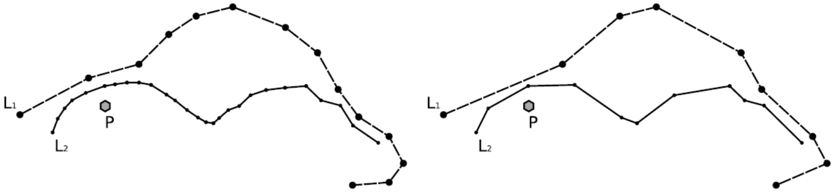
- EPUG-OVERLAY
  - **E**xact.
  - **P**arallel.
  - **U**niform **G**rid.



- PinMesh
  - Exact and efficient point location
  - Point location: subproblem of the mesh overlay



- EPLSimp
  - Map simplification
  - Exact, topologically correct and parallel

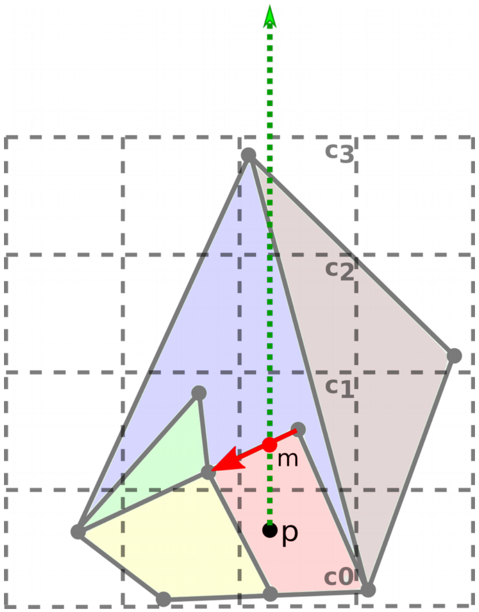


3D Point Location - PinMesh

- Input:
  - A mesh (set of triangles, each one with labels of the region on its positive and negative sides)
  - A set of query points
- Objective determine where the query points are.

3D Point Location - PinMesh

- Idea:
  - Trace a vertical ray from each point
  - Find the lowest triangle above the point
  - Use orientation to locate the point



- Techniques:
  - SoS: no ray hits edges, vertices or starts on triangle.
  - Uniform grid: reduce ray-triangle intersection tests.
  - Parallel programming: grid creation and queries.
  - Rational numbers: exact computation.
- Result: PinMesh is very efficient and robust

2D map overlay algorithm - EPUG-OVERLAY

- Given two polygonal maps, compute their intersection
- Idea:
  - Find all intersections using a uniform grid.
  - Split edges at intersection points.
  - Locate vertices/edges in the other map (using grid).
  - Compute output polygons.

2D map overlay algorithm - EPUG-OVERLAY

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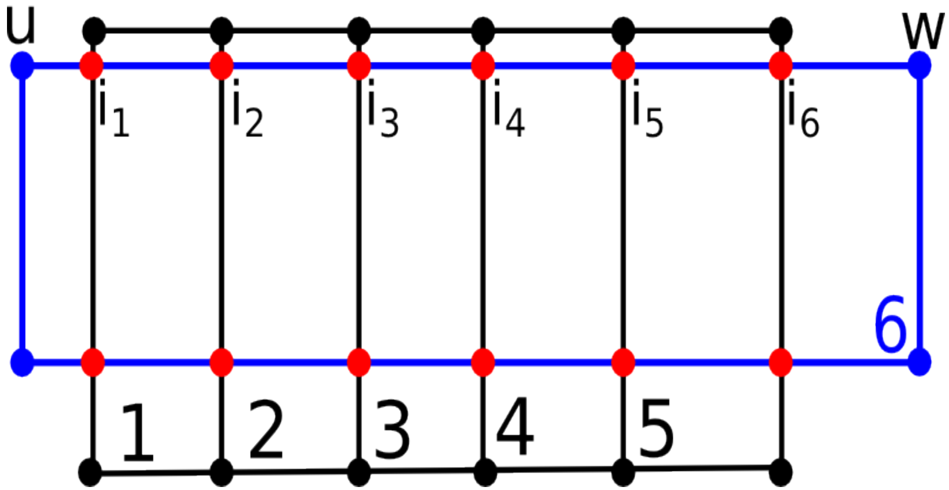

- (u,w) divided into 7 segments.
- 5 will be in output.

2D map overlay algorithm - EPUG-OVERLAY

- Given two polygonal maps, compute their intersection
- Idea:
  - Find all intersections using a uniform grid.
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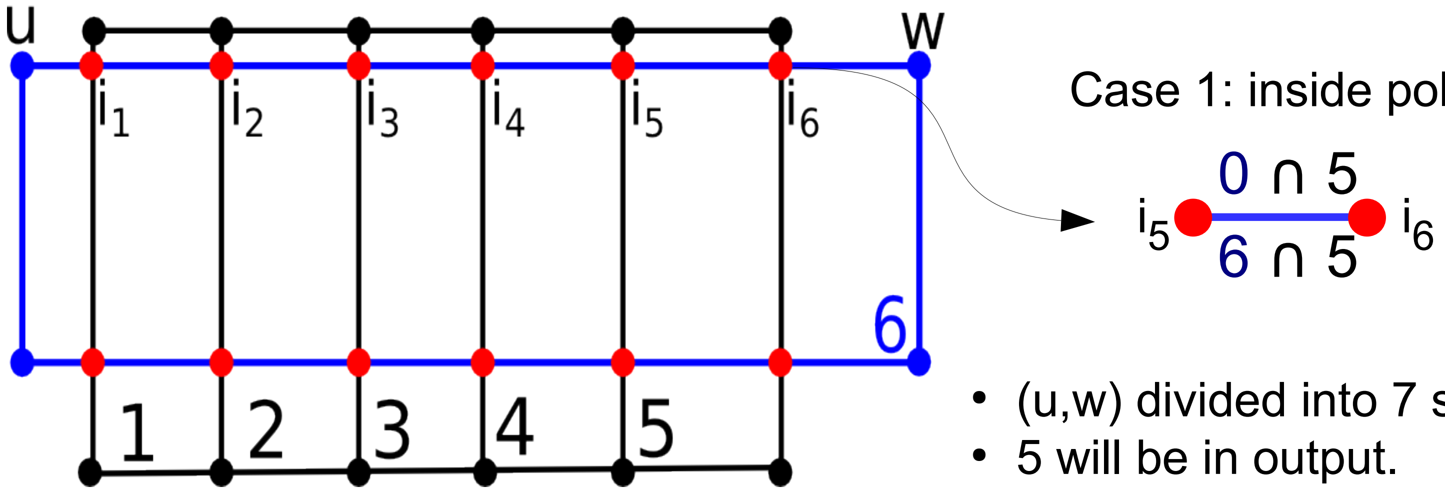
  - **Locate vertices/edges in the other map (using grid).**
  - **Compute output polygons.**



Case 1: inside polygon 5

- (u,w) divided into 7 segments.
- 5 will be in output.

2D map overlay algorithm - EPUG-OVERLAY

- Given two polygonal maps, compute their intersection
- Idea:
  - Find all intersections using a uniform grid.
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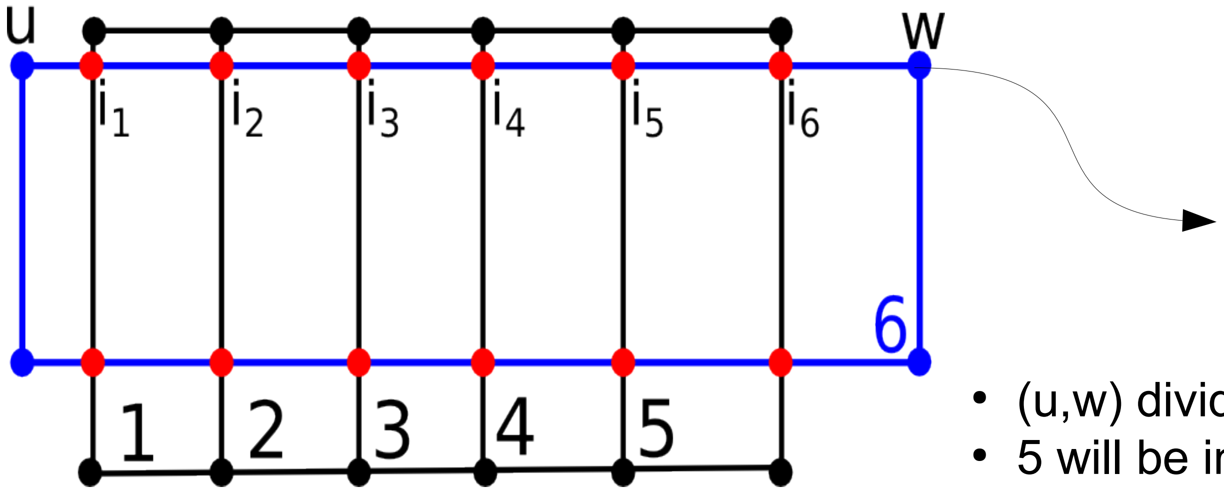
  - **Locate vertices/edges in the other map (using grid).**
  - **Compute output polygons.**



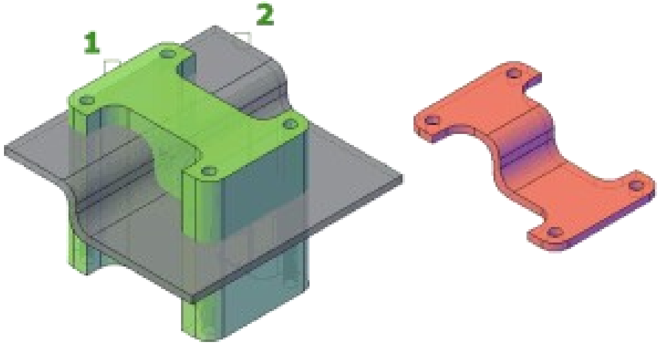
Case 2

(i_6,w) → outside other map

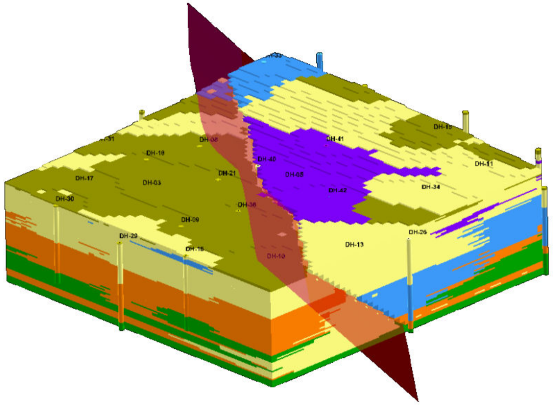
- (u,w) divided into 7 segments.
- 5 will be in output.

Current work: 3D-EPUG-OVERLAY

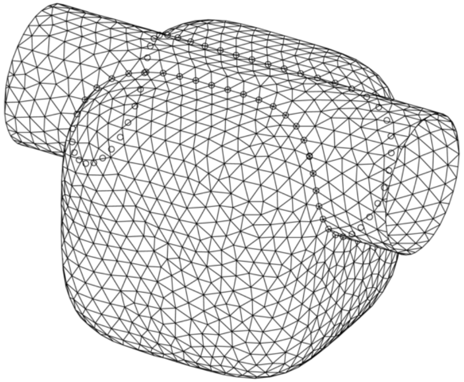
- Apply the same techniques, but for 3D mesh intersection
  - Rational numbers
  - “3D maps” represented by a set of triangles
  - Triangles: left/right objects
  - 3D uniform grid for intersection and point in polygon
  - Simulation of Simplicity
  - Algorithm designed to be **parallel**



Source: Autodesk



Source: Rockworks

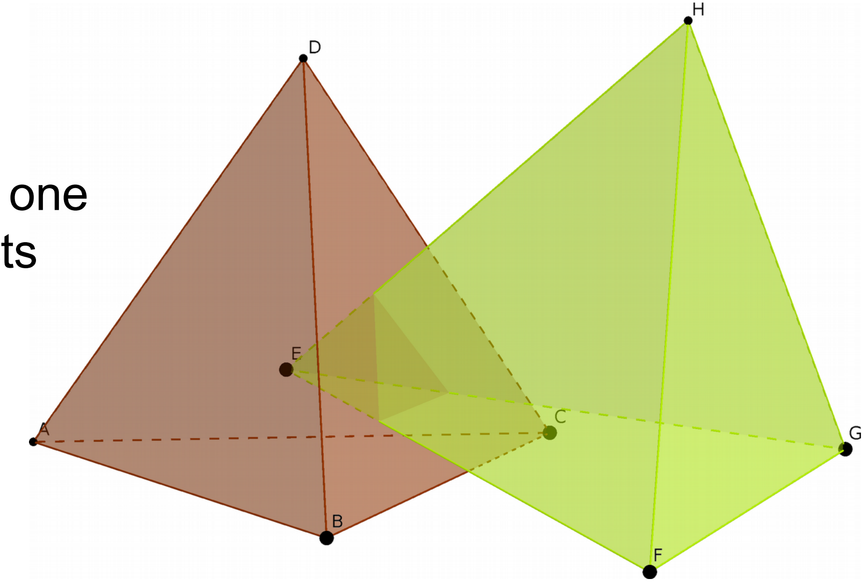


source: wikipedia

First step: triangle-triangle intersections

- A 3D uniform grid is created.
- Triangles from both meshes are inserted into the cells their AABB intersect.
- Cells with “too many” triangles are refined, creating a second level grid.
- Pairs of triangles in each cell are tested for intersection → “Too many” = number of pairs of triangles.
- Intersection tests: Moller's algorithm for performance.
- Cells do not influence each other → process them in parallel

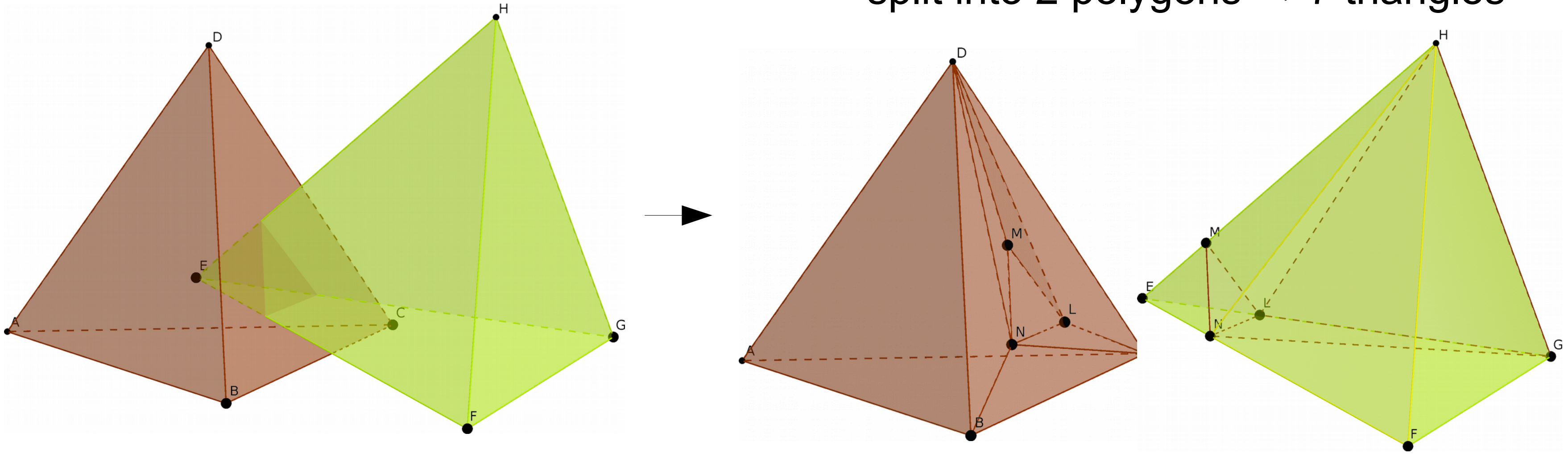
Red mesh: only one triangle intersects green



Second step: retesselation

- Triangles are, then, split at the intersections.
  - Similar to splitting edges in EPUG-OVERLAY.
  - Intersection on each triangle → planar subdivision → retriangulation.
  - Again, this step can be done in parallel on the triangles.

Red intersecting triangle:
split into 2 polygons → 7 triangles



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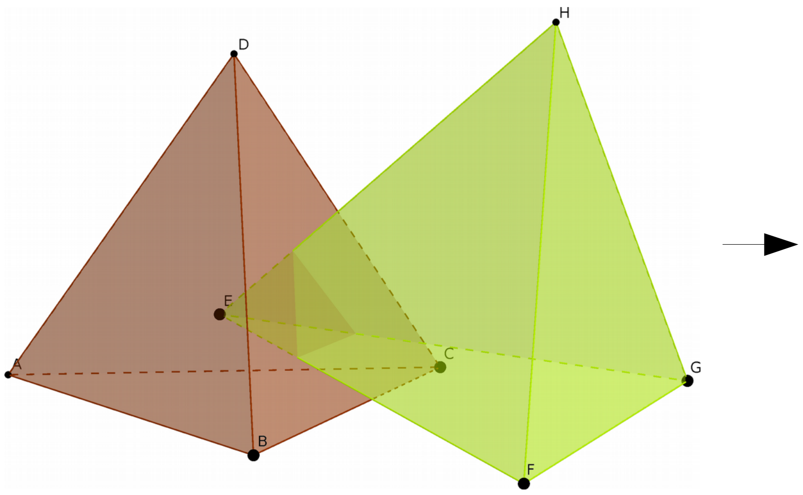
Second step: retesselation

- Retesselated mesh: equivalent to the original
  - Union of each split triangle is equal to the original triangle
  - Non split triangles will also be in retesselated mesh

- After retesselation: intersections will only happen at common vertices/edges.



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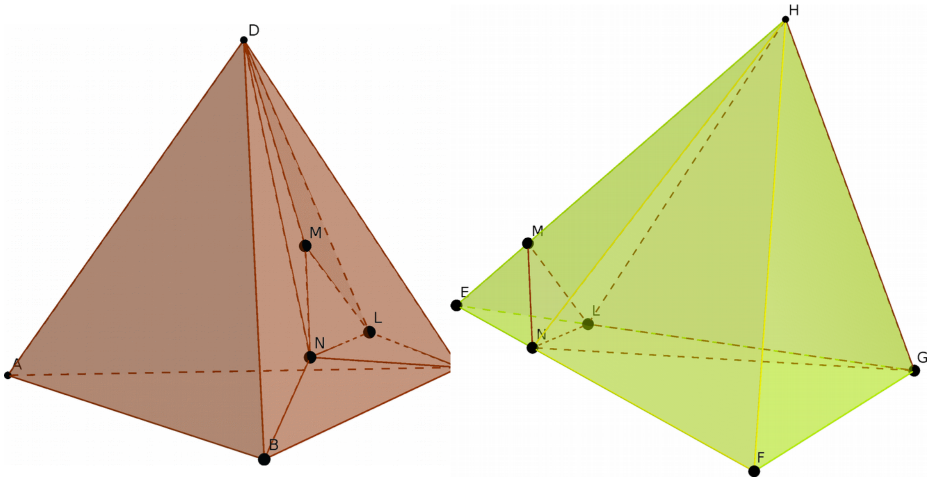
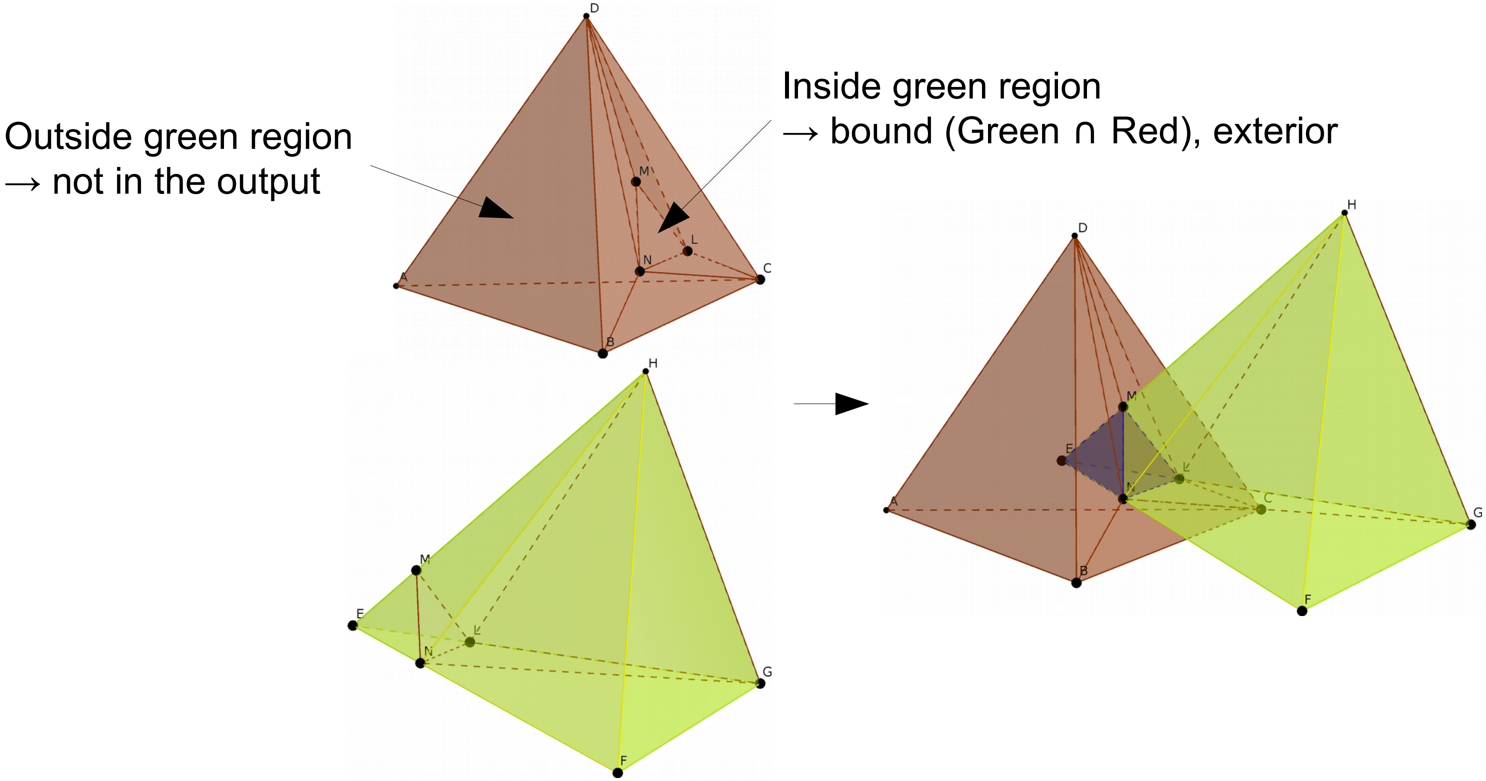


Third step: classification

- Finally, triangles are classified.
  - Similar to edge classification in EPUG-OVERLAY.
- Only two basic cases for each triangle t (bounding A,B):
  - t outside other mesh → t will not be in the output.
  - t inside region R of the other mesh → t will bound R∩A and R∩B.

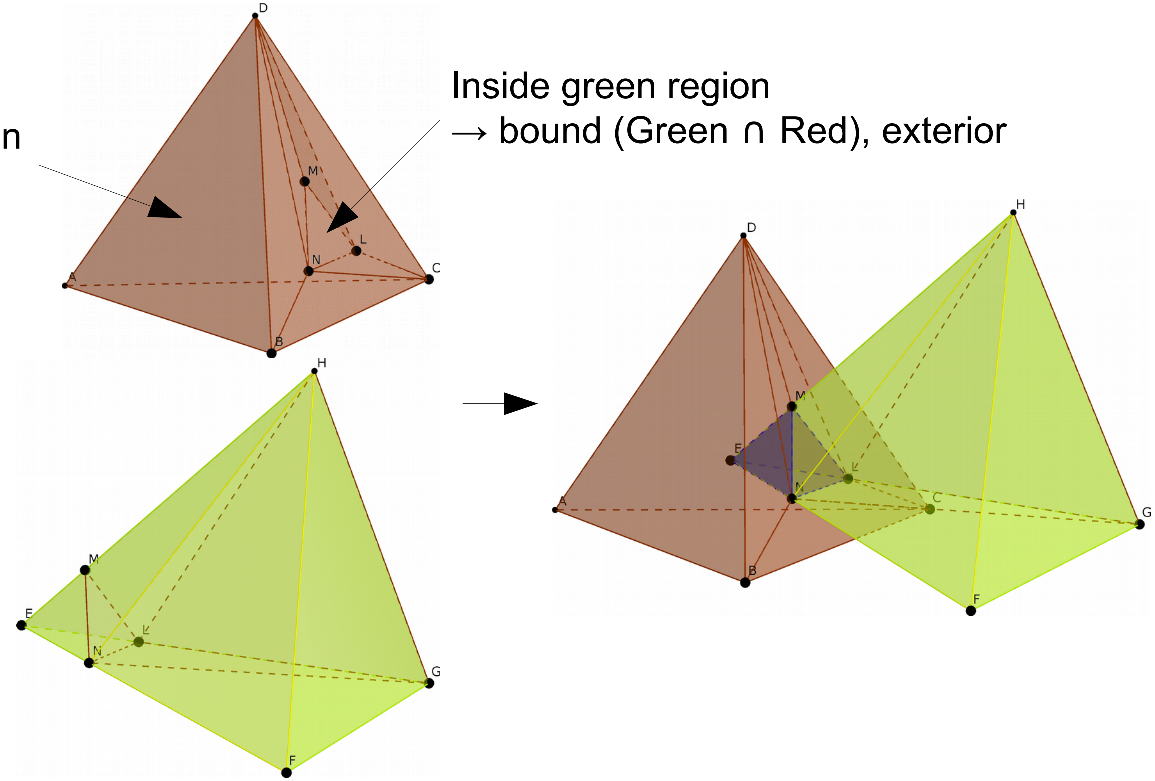


Outside green region
→ not in the output

Inside green region
→ bound (Green ∩ Red), exterior

Third step: classification

- How to locate a triangle?
  - Simple and fast solution: PinMesh

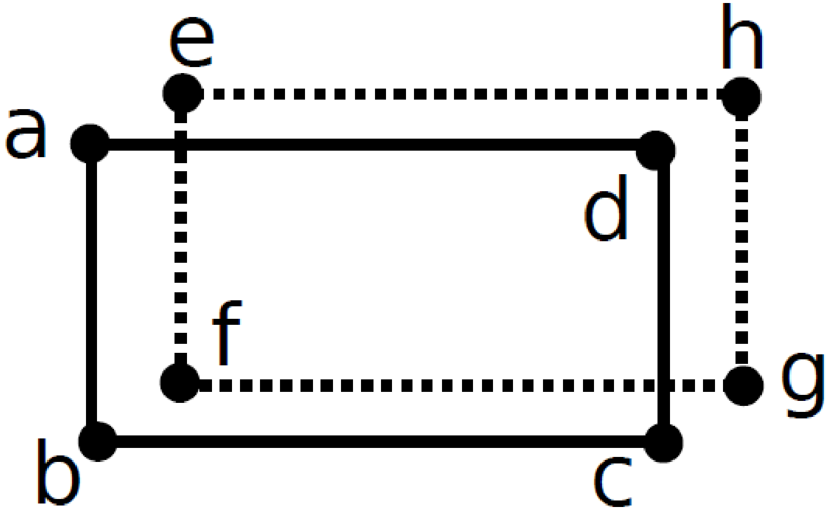


Outside green region
→ not in the output

Inside green region
→ bound (Green ∩ Red), exterior

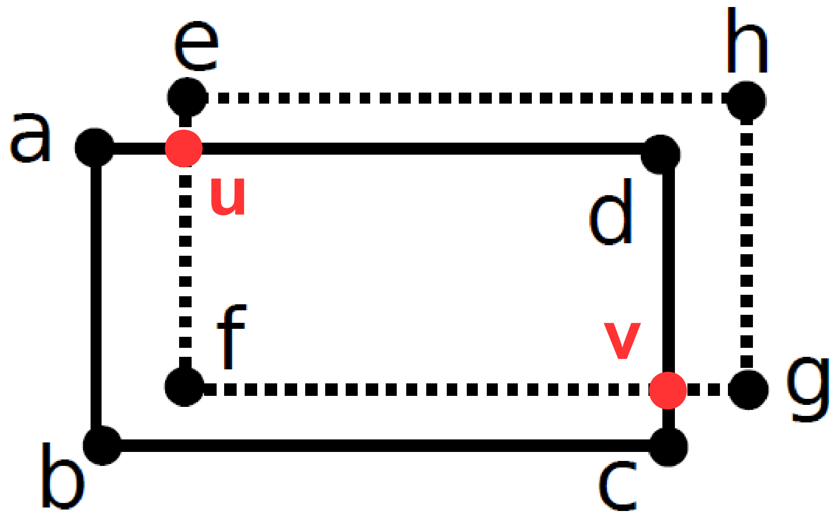
Special cases

- Under development
- Proposed solution: SoS
- SoS was successfully employed in EPUG-OVERLAY
  - Idea: translate one of the maps by $(\varepsilon,\varepsilon^2)$ → no common edges/intersection at endpoints
- Example: two coincident polygons → translation $(\varepsilon,\varepsilon^2)$ → non coincident



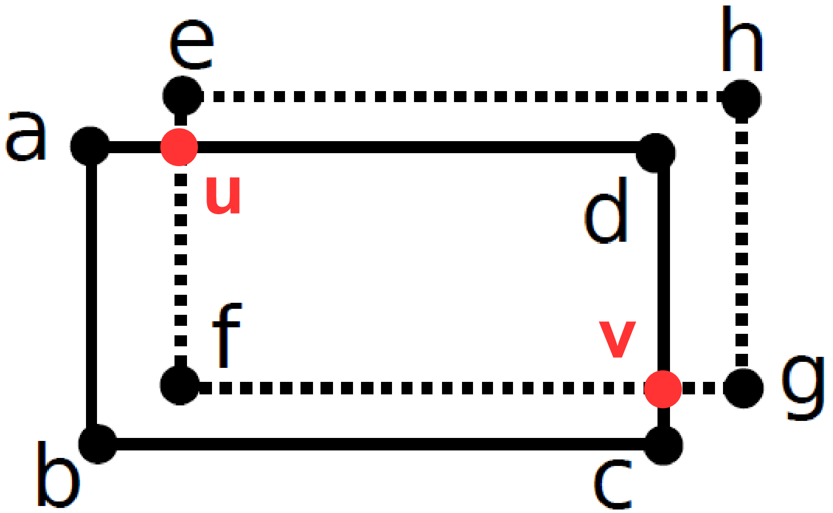
Special cases

- Example: two coincident polygons → translation $(\varepsilon,\varepsilon^2)$ → non coincident
  - **Intersection** computation: two intersections *u* and *v*



Special cases

- Example: two coincident polygons → translation $(\varepsilon,\varepsilon^2)$ → non coincident
  - Intersection computation: two intersections *u* and *v*
  - **Retesselation**: a-d split into a-u, u-d
  - Classification:
    - a-u is outside the other polygon → not in output
    - u-d is inside the other polygon → u-d in the output
      - u-d will bound the interior and the exterior of the output polygon



Special cases

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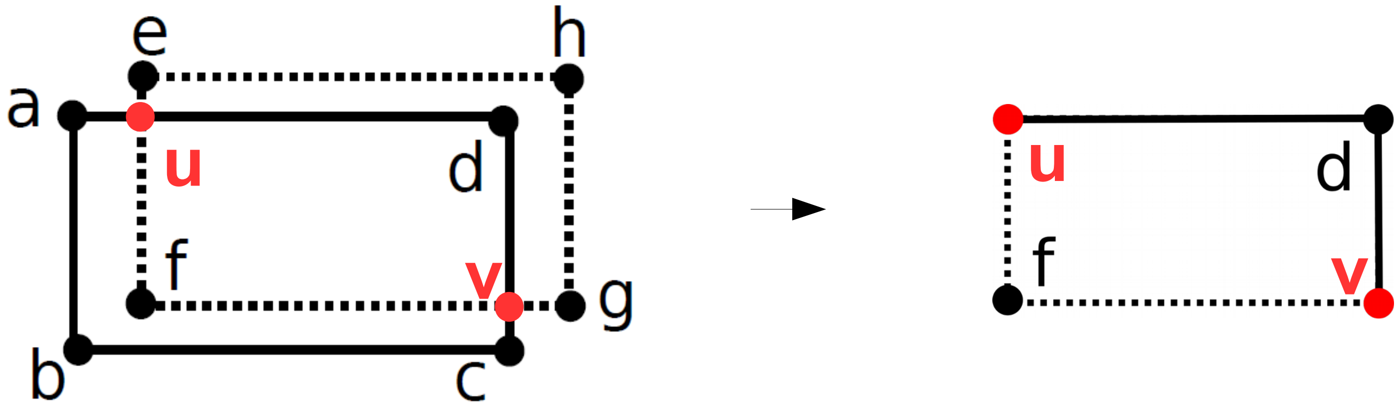
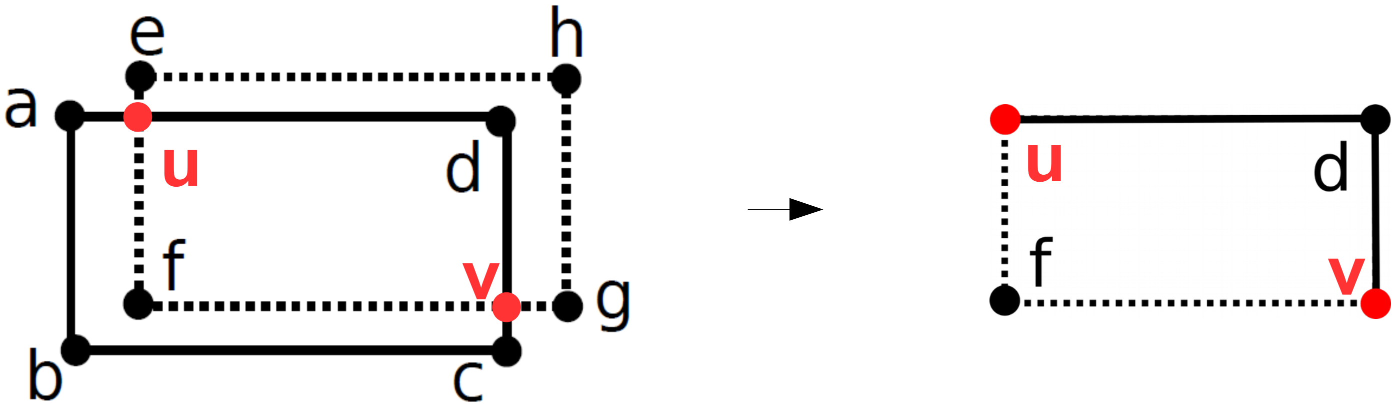


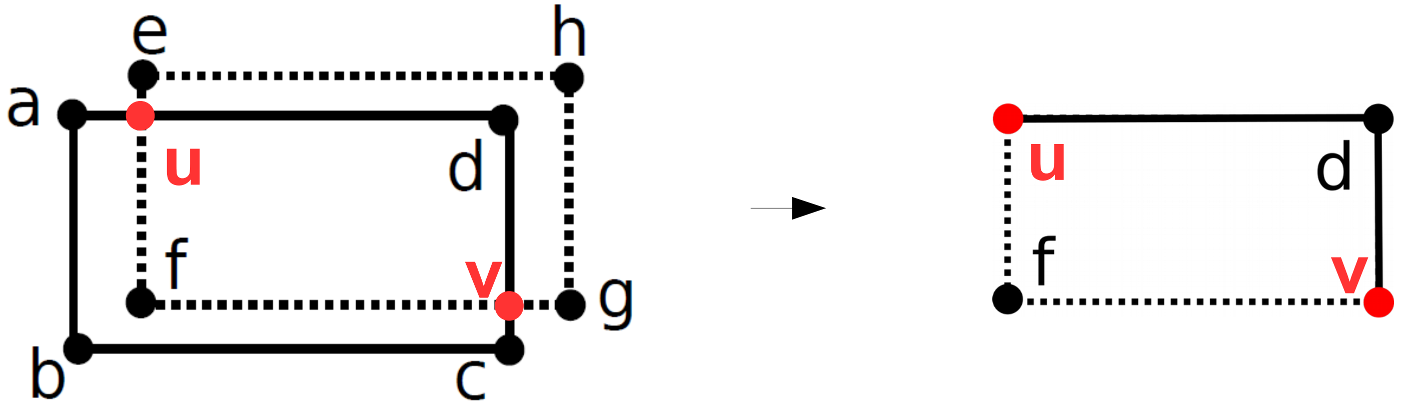
Special cases

- Example: two coincident polygons → translation $(\varepsilon,\varepsilon^2)$ → non coincident
  - But… a-b-c-d is equal to e-f-g-h
  - u-f-v-d should also represent the same polygon!
  - The translation is only conceptual! It only affects the conditionals
  - u=a=e and v=c=g → the polygons are the same!



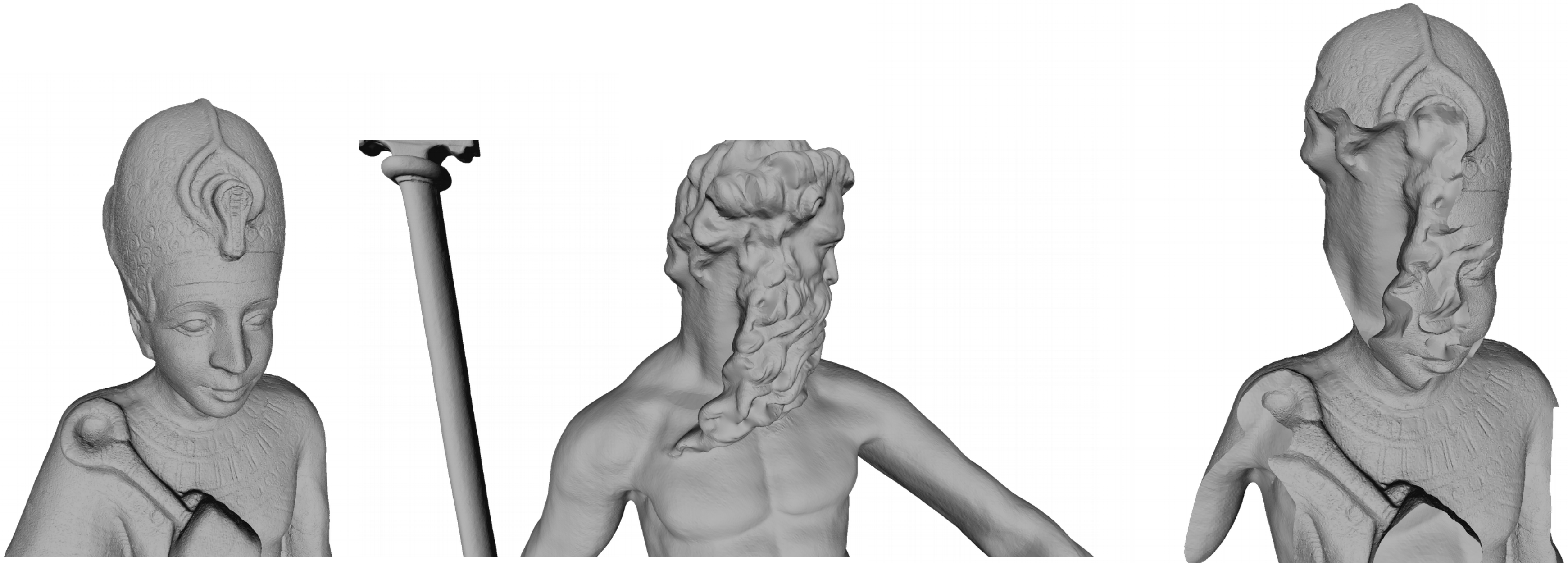
Special cases

- In PinMesh we also employ a similar idea: all the query points are translated by $(\varepsilon,\varepsilon^2,\varepsilon^3)$
- We believe this same perturbation scheme will be suitable for intersecting 3D meshes.



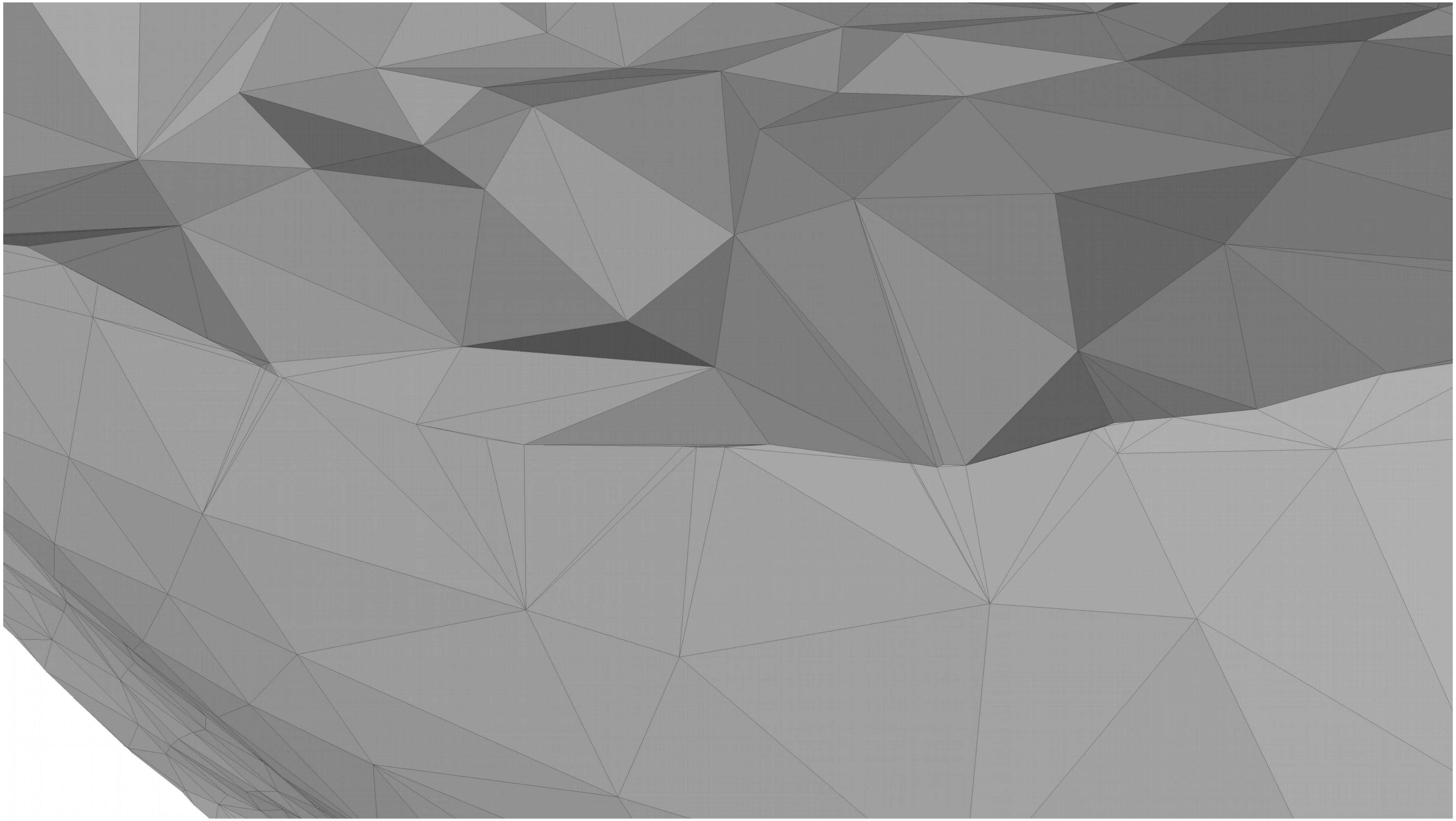
Example of result

- Intersection of two big meshes from AIM@SHAPE:
  - Ramesses: 1.7 million triangles
  - Neptune: 4 million triangles



Example of result

- Hard to process triangles → roundoff errors



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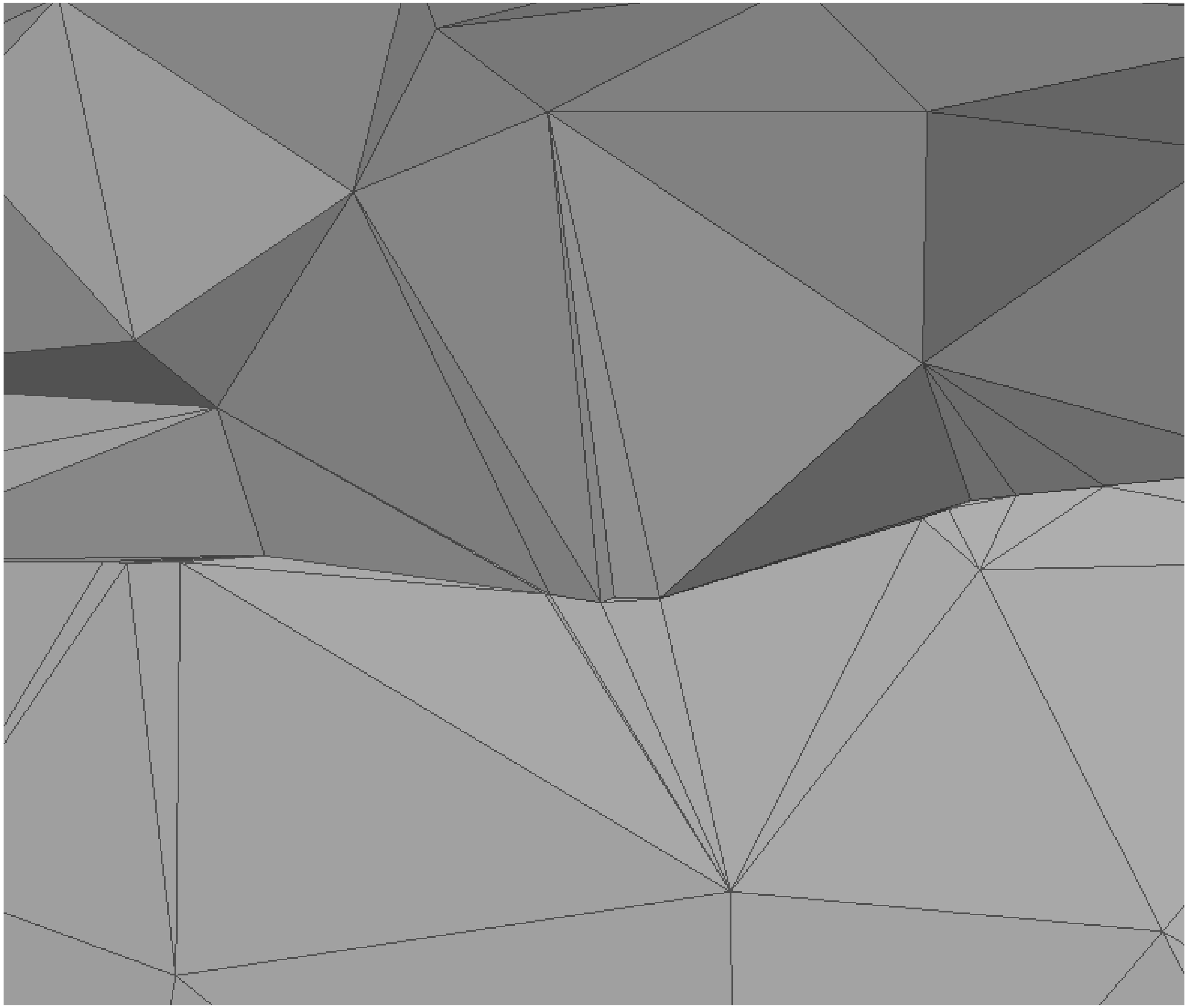

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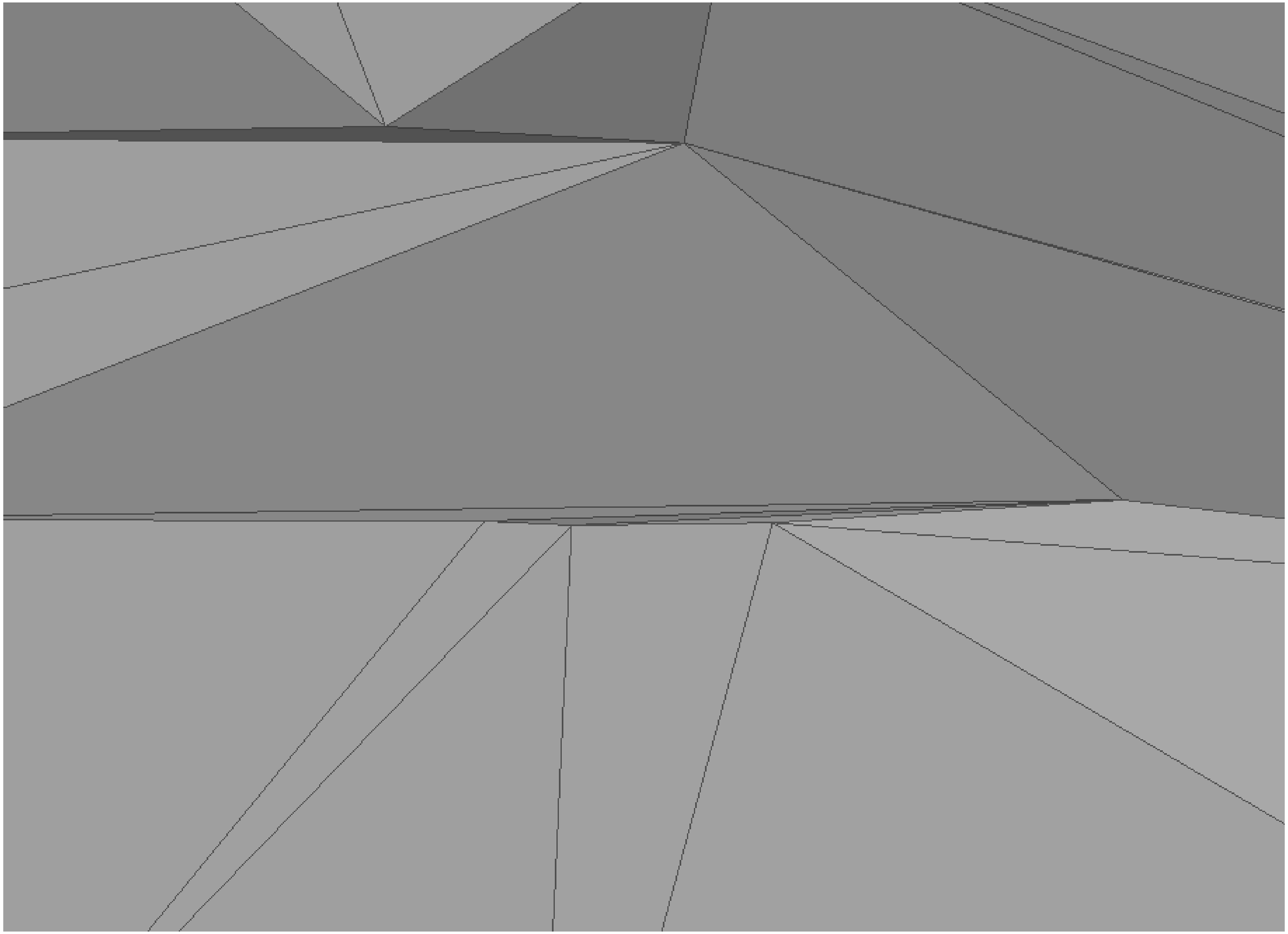
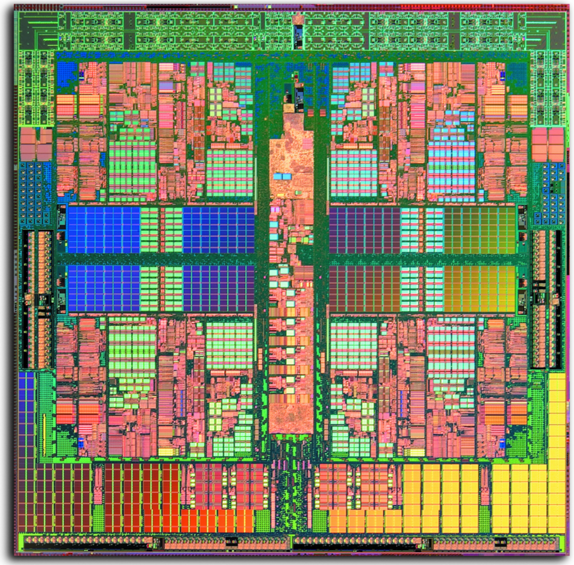


Current work

- Employ techniques successfully applied in our previous works.

- This algorithm →few data dependency →very parallelizable.
  - Uniform grid creation: edges in parallel.
  - Locate vertices in polyhedra.
  - Compute intersections: cells in parallel.
  - Compute output triangles: process input triangles in parallel.

- Most of computers: multicore → OpenMP.

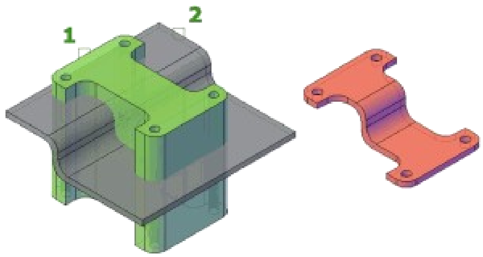


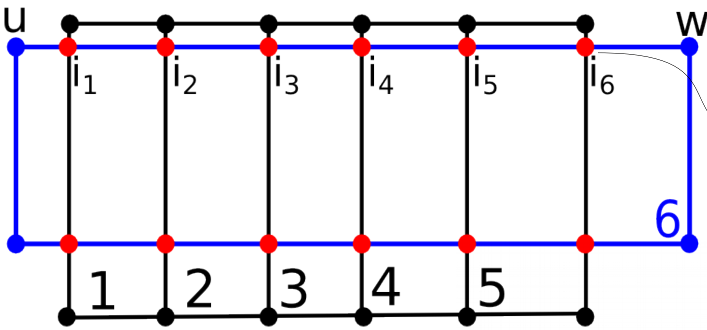
source: wikipedia

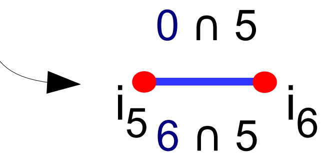
Conclusions

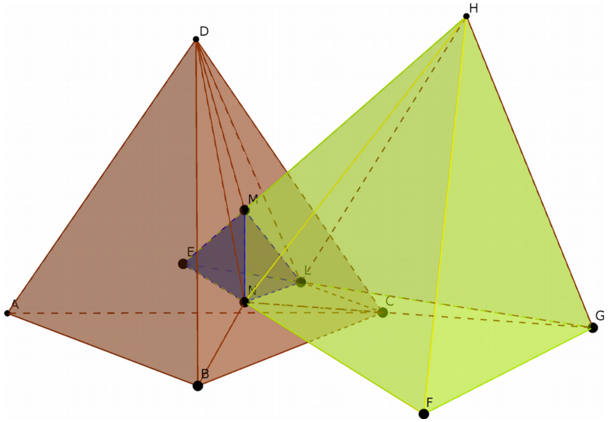
- 3D-EPUG-OVERLAY
  - Exact
  - Parallel
  - Uniform grid
- Part of a bigger project
  - Exact and parallel geometric algorithms
  - Applications in GIS, CAD and AM
- Ongoing/future work
  - SoS perturbation scheme
  - Code optimizations
  - Application of these ideas to other algorithms

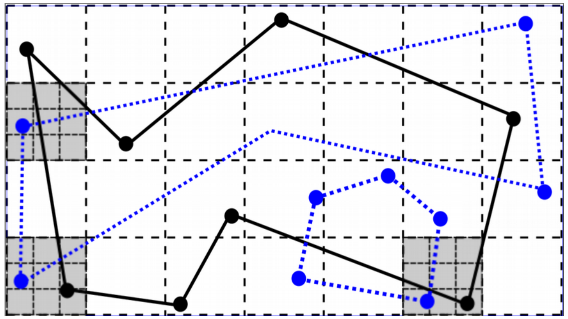
Thank you!

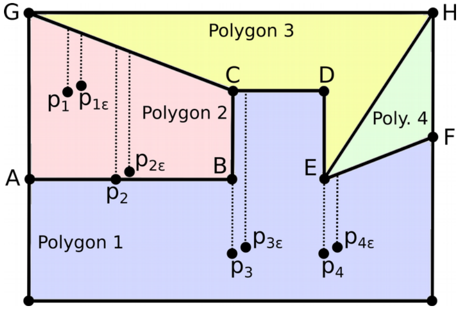


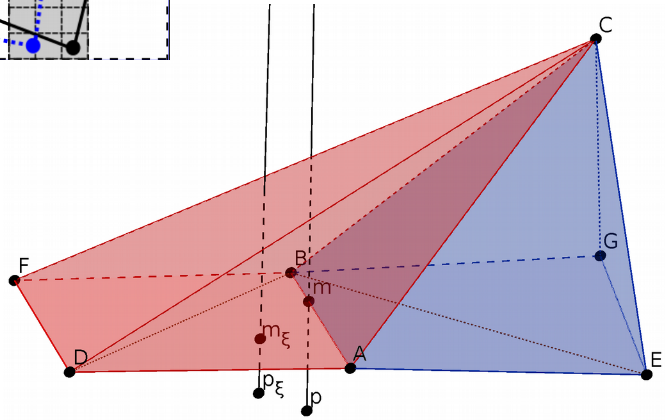












Acknowledgement:

CAPES

Simulation of Simplicity

- Example: how to check if a point *q* is directly “below” the interior of a triangle *t*?
- Project *q* and *t* to z=0, check if *q'* is inside *t'* (also check q_z).
- Is *q' inside t'* ? → barycentric coordinates → $0 < \lambda_i < 1$ for i=1,2 and 3 ?

$$\lambda_0 = \frac{(t'_{1y} - t'_{2y}) \times (q'_x - t'_{2x}) + (t'_{2x} - t'_{1x}) \times (q'_y - t'_{2y})}{det}$$

$$\lambda_1 = \frac{(t'_{2y} - t'_{0y}) \times (q'_x - t'_{2x}) + (t'_{0x} - t'_{2x}) \times (q'_y - t'_{2y})}{det}$$

$$\lambda_2 = 1 - \lambda_0 - \lambda_1$$

$$det = (t'_{1y} - t'_{2y}) \times (t'_{0x} - t'_{2x}) + (t'_{2x} - t'_{1x}) \times (t'_{0y} - t'_{2y})$$

Simulation of Simplicity

- Degeneracies: det = 0 → vertical triangle
- Point on boundary of t' ($\lambda_i = 0$ *or* 1).

- SoS → $q(x,y,z) \rightarrow q_\varepsilon(x+\varepsilon,y+\varepsilon^2,z+\varepsilon^3)$, $q'(x,y) \rightarrow q'_\varepsilon(x+\varepsilon,y+\varepsilon^2)$
  - q'_ε will never be on a vertex or edge of t'.
    - q' is not on vertex/edge → q'_ε is also not on vertex/edge (infinitesimal).
    - q' is on vertex/edge → q'_ε is not on vertex/edge (infinitesimal/slope).
      - Ex: q' is on an edge → q'_ε cannot be on the same edge (slope would be infinitesimal)

Simulation of Simplicity

- SoS implementation:
  - q'_{ε} will never be on a vertex or edge of t' → if det=0 → false
  - Replace q' with q'_{ε}

$$\lambda_0 = \frac{(t'_{1y} - t'_{2y}) \times (q'_x - t'_{2x}) + (t'_{2x} - t'_{1x}) \times (q'_y - t'_{2y})}{det}$$

$$\lambda_1 = \frac{(t'_{2y} - t'_{0y}) \times (q'_x - t'_{2x}) + (t'_{0x} - t'_{2x}) \times (q'_y - t'_{2y})}{det}$$

$$\lambda_2 = 1 - \lambda_0 - \lambda_1$$

$$det = (t'_{1y} - t'_{2y}) \times (t'_{0x} - t'_{2x}) + (t'_{2x} - t'_{1x}) \times (t'_{0y} - t'_{2y})$$

Simulation of Simplicity

- SoS implementation:
  - q'_ε will never be on a vertex or edge of t' → if det=0 → false
  - Replace q' with q'_ε → λ_i with $\lambda_{\varepsilon i}$
  - E.g.: is $0 < \lambda_{\varepsilon 0}$?
    - $\lambda_0 \neq 0 \rightarrow$ check λ_0
    - $\lambda_0 = 0 \rightarrow$ check $t'_{1y} - t'_{2y}$
    - $t'_{1y} - t'_{2y} = 0 \rightarrow$ check $t'_{2x} - t'_{1x}$
    - Both can't be 0.

$$\lambda_{\epsilon_0} = \lambda_0 + \frac{(t'_{1y} - t'_{2y}) \times \epsilon + (t'_{2x} - t'_{1x}) \times \epsilon^2}{det}$$

$$\lambda_{\epsilon_1} = \lambda_1 + \frac{(t'_{2y} - t'_{0y}) \times \epsilon + (t'_{0x} - t'_{2x}) \times \epsilon^2}{det}$$

$$\lambda_{\epsilon_2} = 1 - \lambda_{\epsilon_0} - \lambda_{\epsilon_1}$$