9/22/16 p1

TO COMBINE 2 ROTATIONS

1: 180° ABOUT x AXIS

2 180° ABOUT y AXIS

$$q = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}(A_x i + A_y j + A_z k)$$

1: $\theta = 180°$ $\cos\frac{\theta}{2} = \cos 90° = 0$.

$\sin\frac{\theta}{2} = 1$

$q_1 = i$

ex $p = (1,2,3)$ $\quad i + 2j + 3k$

$p' = q p q^*$ $= i(i + 2j + 3k)(-i)$

$= (i^2 + 2ij + 3ik)(-i)$

$= (-1 + 2k - 3j)(-i)$

$= i - 2ki + 3ji$ $= i - 2j - 3k$

$(1, -2, -3)$

R_1 180° ABOUT X AXIS

$q_1 = i$

R_2 = 180° ABOUT Y AXIS

$q_2 = j$

COMBINE R_1 THEN R_2

$R = q_2 q_1 = ji = -k$

$-k = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}(k)$

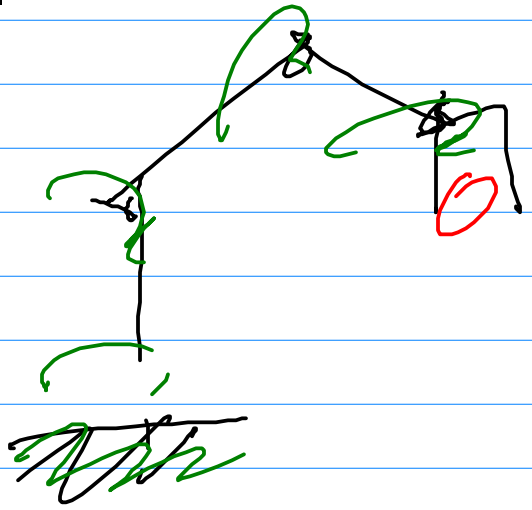
$\cos\frac{\theta}{2} = 0$; $\sin\frac{\theta}{2} = 1$

AXIS = (0, 0, 1) $\frac{\theta}{2} = 90°$ $\theta = 180°$

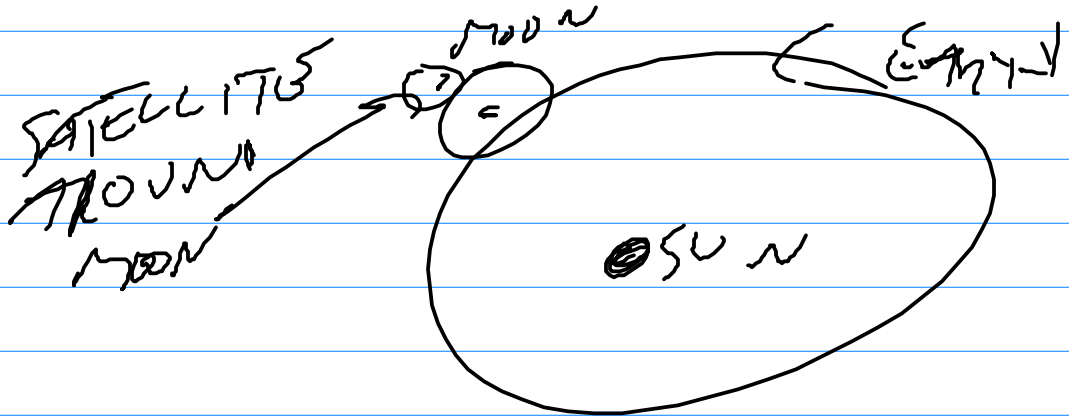
COMBINING 180° ABOUT X AXIS THEN 180° ABOUT Y AXIS IS SAME AS 180° ABOUT Z AXIS

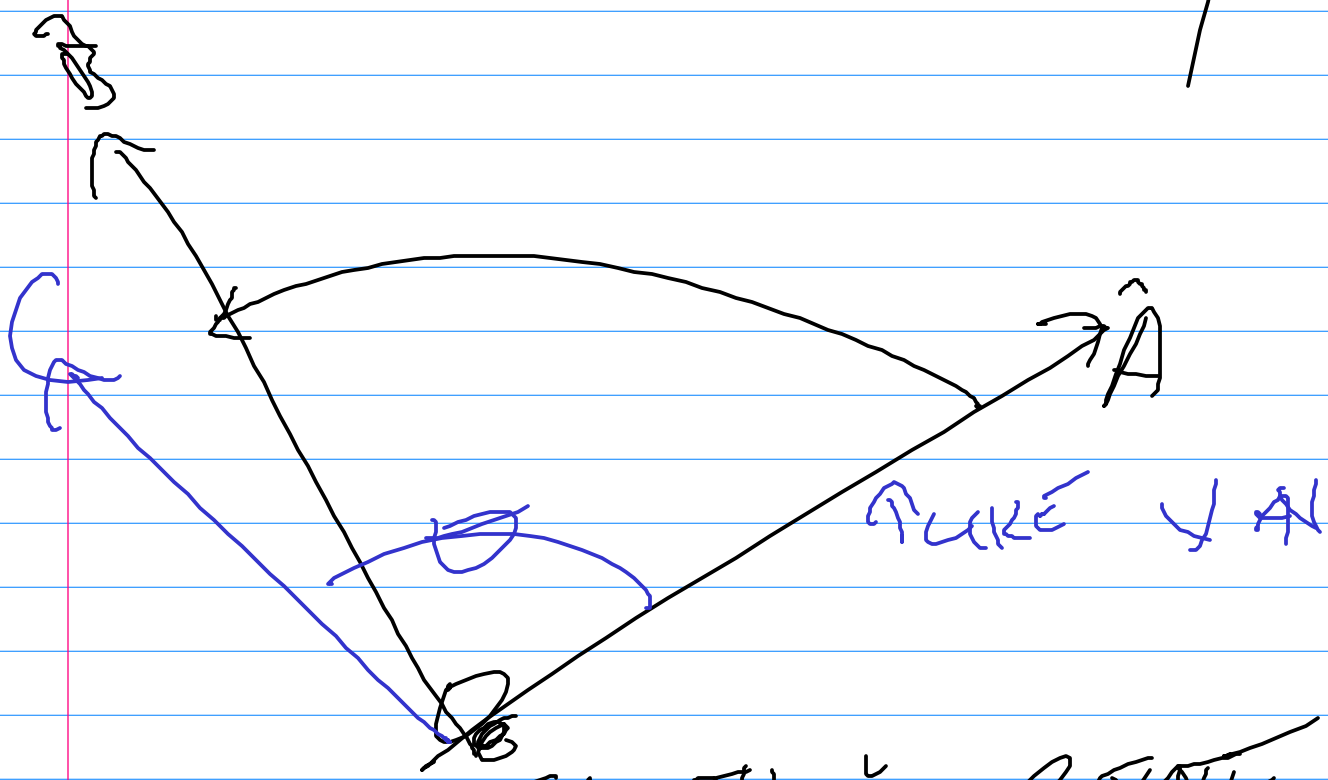
Q Why would you want to combine rotations?

A1: robot arm.



A2 spacecraft





WANT TO SMOOTHLY ROTATE FROM A TO B.

$C = A \times B$

NORMALIZE C $\quad C = \frac{C}{|C|}$

ANGLE BETWEEN A, B $= \theta$

$A \cdot B = \cos\theta$

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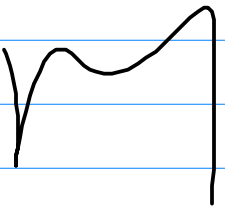
TO ANIMATE 3D ROTATION W MATRIX



WANT TO DO M IN 100 STEPS

NEED $M^{1/100}$

CAN DO MATRIX ROOT WITH EIGENVECTORS + EIGENVALUES.

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$q = i$ WHAT ROTATION?

WHAT AXIS θ?

$$q = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}(a_1 i + a_2 j + a_3 k)$$

$\cos\frac{\theta}{2} = 0$

$\sin\frac{\theta}{2} = 1$ $\quad a = (1, 0, 0)$

$\frac{\theta}{2} = 90^\circ$ $\quad \theta = 180^\circ$.